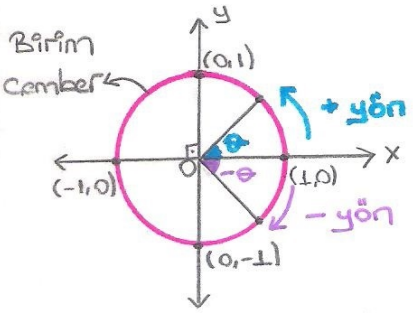
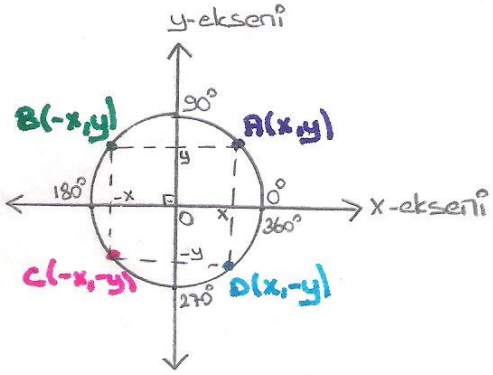
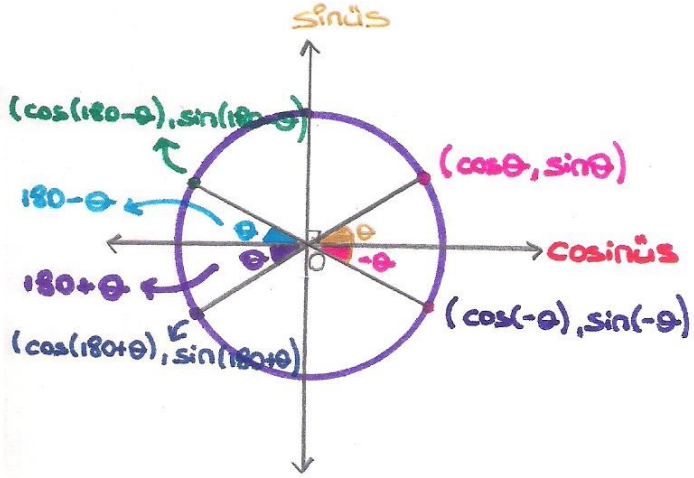
~ Trigonometri ~





Not: Trigonometride x-eks → cosinüs
y-eks → sinüs

olduğundan bu noktaları başlangıç noktasına birleştirdiğimizde θ açı alırsa, yeni koordinatlar;



Esas ölçü: Açının 0° ile 360° arasındaki ölçüsüne denir.

Not: $\pi = 180° \rightarrow \frac{\pi}{2} = 90°$, $2\pi = 360°$

1. Durum: 360° den büyük bir açı verildiğinde esas ölçünün bulunması:

Örnek: 1970° nin esas ölçüsü nedir?

Çözüm: Verilen sayı 360° ye bölünür, kalan sayı esas ölçüyü verir.

not: Bölme işlemi yapılırken en sondaki sıfırlar sadeleştirilemez.

$$\begin{array}{r|l} 1970 & 360 \\ -1800 & 5 \\ \hline 170 & \end{array}$$

esas ölçü: 170°

2. Durum: 0° den küçük olan bir açı verildiğinde esas ölçünün bulunması:

Örnek: −3900° nin esas ölçüsü nedir?

Çözüm: Sayı pozitif düşünülerek 360° ye bölünür, kalan sayı 360° den çıkarılırsa sonuç esas ölçüyü verir.

$$\begin{array}{r|l} 3900 & 360 \\ -360 & 10 \\ \hline 300 & \end{array} \qquad \begin{array}{r} 360 \\ -300 \\ \hline 60° \end{array}$$

esas ölçü: 60°

3. Durum: 2π den büyük bir açı verildiğinde esas ölçünün bulunması:

Örnek: $\frac{19\pi}{3}$ radyanın esas ölçüsü nedir?

Çözüm: Sayının yaklaşık değeri bulunur ve içinden $2\pi, 4\pi, 6\pi, 8\pi \ldots$ gibi katlar çıkarılır. $\frac{19}{3}\pi \cong 6,\ldots\pi \rightarrow$ en fazla 6π atılır.

$\frac{19\pi}{3} - 6\pi = \frac{\pi}{3}$ → **esas ölçü: $\frac{\pi}{3}$**

Örnek: $\frac{29\pi}{5}$ radyanın esas ölçüsü nedir?

Çözüm: $\frac{29\pi}{5} \cong 5,...\pi$ en fazla 4π atılır.

$\frac{29\pi}{5} - 4\pi = \frac{9\pi}{5}$ → **esas ölçü: $\frac{9\pi}{5}$**

4. Durum: 0° den küçük radyan türünden bir açı verildiğinde esas ölçüsünün bulunması:

Örnek: $-\frac{33\pi}{5}$ radyanın esas ölçüsü nedir?

Çözüm: Sayının yaklaşık değeri bulunur ve sayıyı pozitif yapacak en küçük $2\pi, 4\pi, 6\pi, 8\pi....$ açı eklenir.

$-\frac{33\pi}{5} \cong -6,...\pi$ → en az 8π eklenmeli

$-\frac{33\pi}{5} + 8\pi = \frac{7\pi}{5}$ → **esas ölçü: $\frac{7\pi}{5}$**

Örnek: $-\frac{29\pi}{5}$ radyanın esas ölçüsü nedir?

Çözüm: $-\frac{29\pi}{5} \cong -5,...\pi$ → en az 6π eklenmeli

$-\frac{29\pi}{5} + 6\pi = \frac{\pi}{5}$ → **esas ölçü: $\frac{\pi}{5}$**

Alıştırmalar:

Aşağıda verilen açıların esas ölçülerini bulunuz?

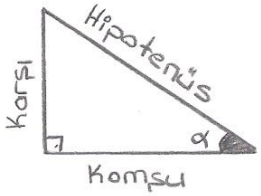
1) 2007° → **C: 207°**

2) -220° → **C: 140°**

3) $\frac{57\pi}{5}$ → **C: $\frac{7\pi}{5}$**

4) $-\frac{46\pi}{3}$ → **C: $\frac{2\pi}{3}$**

Trigonometrik Fonksiyonlar



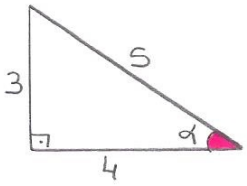
$\sin\alpha = \frac{\text{Karşı}}{\text{Hip.}}$ $\quad \tan\alpha = \frac{\text{Karşı}}{\text{Komşu}}$

$\cos\alpha = \frac{\text{Komşu}}{\text{Hip.}}$ $\quad \cot\alpha = \frac{\text{Komşu}}{\text{Karşı}}$

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ $\quad \sec\alpha = \frac{1}{\cos\alpha}$

$\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$ $\quad \operatorname{cosec}\alpha = \frac{1}{\sin\alpha}$

Örnek:



$\frac{\sin\alpha.\cos\alpha + \tan\alpha}{\cot\alpha} = ?$

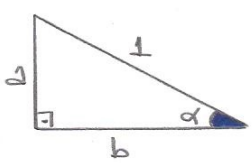
Çözüm: $\sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$

$\tan\alpha = \frac{3}{4}$, $\cot\alpha = \frac{4}{3}$

$\frac{\sin\alpha.\cos\alpha + \tan\alpha}{\cot\alpha} = \frac{\frac{3}{5}.\frac{4}{5} + \frac{3}{4}}{\frac{4}{3}} = \frac{\frac{12}{25} + \frac{3}{4}}{\frac{4}{3}} \quad \begin{matrix}(4) \\ (25)\end{matrix}$

$\frac{123}{100} . \frac{3}{4} = \frac{369}{400}$

Özellik:



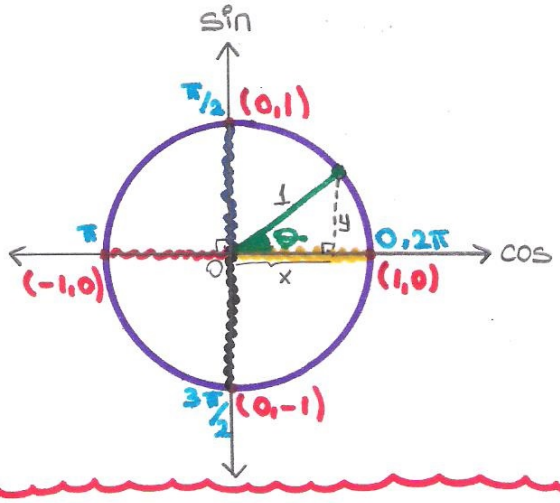
$\sin\alpha = \frac{a}{1}$

$\cos\alpha = \frac{b}{1}$

$a^2 + b^2 = 1$

$\sin^2\alpha + \cos^2\alpha = 1$

- $\sin^2 15 + \cos^2 15 = 1$
- $\sin^2 1970 + \cos^2 1970 = 1$
- $\sin^2 \frac{71\pi}{5} + \cos^2 \frac{71\pi}{5} = 1$
- $\sin^2\left(\frac{-73\pi}{17}\right) + \cos^2\left(\frac{-73\pi}{17}\right) = 1$



Cosθ → Açı θ° derecedeyken doğrunun çemberi kestiği noktanın x değeri

Sinθ → Açı θ° derecedeyken doğrunun çemberi kestiği noktanın y değeri

$\cos 0 = 1$	$\sin 0 = 0$
$\cos \frac{\pi}{2} = \cos 90 = 0$	$\sin \frac{\pi}{2} = \sin 90 = 1$
$\cos \pi = \cos 180 = -1$	$\sin \pi = \sin 180 = 0$
$\cos \frac{3\pi}{2} = \cos 270 = 0$	$\sin \frac{3\pi}{2} = \sin 270 = -1$
$\cos 2\pi = \cos 360 = 1$	$\sin 2\pi = \sin 360 = 0$

$$-1 \leq \cos\theta \leq 1$$
$$-1 \leq \sin\theta \leq 1$$

Sonuç: $\cos\theta$ ve $\sin\theta$ → -1 den küçük olamaz ve 1 den büyük olamaz.

$\sin x = 3 \longrightarrow$ Ç.K $= \emptyset$

$\sin 27x = 4 \longrightarrow$ Ç.K $= \emptyset$

$\cos(-1273x^2) = -5 \longrightarrow$ Ç.K $= \emptyset$

Örnek: $x \in \mathbb{R}$, $\cos\alpha = \frac{2x-6}{3}$ ise x hangi aralıktadır?

Çözüm:

$-1 \leq \cos\alpha \leq 1$

$-1 \leq \frac{2x-6}{3} \leq 1$, $-3 \leq 2x-6 \leq 3$

$3 \leq 2x \leq 9$, $\frac{3}{2} \leq x \leq \frac{9}{2}$

Örnek: $\frac{5\sin x + 3}{2}$ ifadesinin alacağı kaç farklı tamsayı değeri vardır?

Çözüm: $-1 \leq \sin x \leq 1$

$-5 \leq 5\sin x \leq 5$, $-2 \leq 5\sin x + 3 \leq 8$

$-1 \leq \frac{5\sin x + 3}{2} \leq 4$

$\{-1, 0, 1, 2, 3, 4\}$ → 6 tane

Örnek: $2\sin^2 x + 5 = a$ ise a hangi aralıkta değer alır?

Çözüm: $-1 \leq \sin x \leq 1$

$0 \leq \sin^2 x \leq 1$, $0 \leq 2\sin^2 x \leq 2$

$5 \leq 2\sin^2 x + 5 \leq 7$, $[5,7]$

Örnek: $(\cos x+7).(5-\cos x)$ çarpımının en büyük değeri kaçtır?

Çözüm: Çarpımın en büyük olması için sayıların birbirine en yakın olması gerekir. "$\cos x=-1$" için $6.6=36$

2010-LYS: $f:\mathbb{R}\to\mathbb{R}$

$$f(x)=\begin{cases}2\sin x, & \sin x\geq 0\\ 0, & \sin x<0\end{cases}$$

Buna göre $(-\pi,\pi)$ aralığının f altındaki görüntüsü nedir?

Çözüm: $(-\pi,\pi)\to(-180,180)$ aralığında $\sin x=1$ en fazla Dolayısıyla, $2\sin x=2$ olabilir sinüsün negatif değerleri için en az "0" olabildiğine göre, $[0,2]$

1966-ÜSS: $\sin 2x=m$ ise m hangi aralıktadır?

Çözüm: $-1\leq\sin 2x\leq 1$, $-1\leq m\leq 1$

Trigonometrik fonksiyonlarla işlemler:

Örnek: $\dfrac{\sin x+\tan x}{1+\cos x}$ ifadesinin en sade hali?

Çözüm: $\dfrac{\sin x+\frac{\sin x}{\cos x}}{1+\cos x}=\dfrac{\frac{\sin x\cos x+\sin x}{\cos x}}{1+\cos x}$

$=\dfrac{\sin x(\cos x+1)}{\cos x}.\dfrac{1}{1+\cos x}=\dfrac{\sin x}{\cos x}=\tan x$

Örnek: $\dfrac{\tan x-\cot x}{\sec x-\operatorname{cosec} x}$ ifadesinin en sade hali?

Çözüm: $\dfrac{\frac{\sin x}{\cos x}-\frac{\cos x}{\sin x}}{\frac{1}{\cos x}-\frac{1}{\sin x}}=\dfrac{\frac{\sin^2x-\cos^2x}{\sin x\cos x}}{\frac{\sin x-\cos x}{\sin x\cos x}}$

$=\dfrac{(\sin x-\cos x).(\sin x+\cos x)}{\sin x-\cos x}=\sin x+\cos x$

Örnek: $(1+\sin x)(\sec x-\tan x)=?$

Çözüm: $(1+\sin x)\left(\dfrac{1}{\cos x}-\dfrac{\sin x}{\cos x}\right)$

$=\dfrac{(1+\sin x).(1-\sin x)}{\cos x}=\dfrac{1-\sin^2x}{\cos x}=\dfrac{\cos^2x}{\cos x}=\dfrac{\cos x.\cos x}{\cos x}=\cos x$

Not: $\sin^2x+\cos^2x=1$

Örnek: $\dfrac{\tan x}{\sec x-1}-\dfrac{\sin x}{1+\cos x}=?$

Çözüm: $\dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}-\frac{1}{1}}-\dfrac{\sin x}{1+\cos x}=\dfrac{\sin x}{1+\cos x}.\dfrac{\cos x}{1-\cos x}-\dfrac{\sin x}{1+\cos x}$

$=\dfrac{\sin x}{1-\cos x}-\dfrac{\sin x}{1+\cos x}=\dfrac{\sin x+\sin x.\cos x-\sin x+\sin x\cos x}{1-\cos^2x}$

$=\dfrac{2\sin x\cos x}{\sin^2x}=\dfrac{2.\sin x.\cos x}{\sin x.\sin x}=2\cot x$

2010-LYS: $\dfrac{(\sin x-\cos x)^2}{\cos x}+2\sin x=?$

Çözüm: $\dfrac{\sin^2x-2\sin x.\cos x+\cos^2x}{\cos x}+\dfrac{2\sin x}{\frac{1}{\cos x}}$

$\dfrac{1-2\sin x.\cos x}{\cos x}+\dfrac{2\sin x\cos x}{\cos x}=\dfrac{1}{\cos x}$

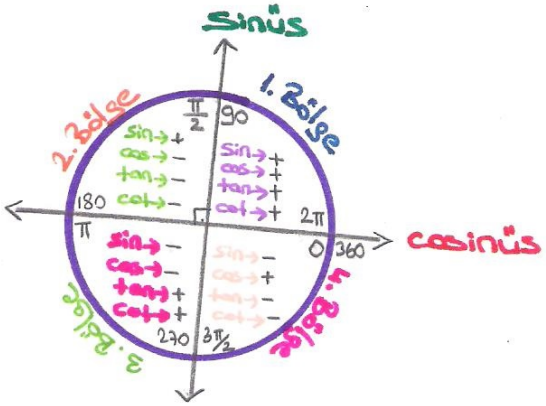
1971-ÜSS: $\frac{\cos^2\theta}{1-\sin\theta}$ ifadesinin en sade hali?

Çözüm: $\frac{1-\sin^2\theta}{1-\sin\theta} = \frac{(1-\sin\theta)(1+\sin\theta)}{1-\sin\theta} = 1+\sin\theta$

Trigonometrik Fonk. İşaretleri:

x ekseni ⟶ cosinüs

y ekseni ⟶ sinüs



	1	2	3	4
sinüs	+	+	−	−
cos	+	−	−	+
tan	+	−	+	−
cot	+	−	+	−

1981-ÖYS: sin85, tan175, cos260, cot275 trigonometrik fonk. işaretlerini bulunuz?

Çözüm:

sin 85° → 1. bölgede → 1. bölgede sinüs → +

tan 175° → 2. bölgede → 2. bölgede tan → −

cos 260° → 3. bölgede → 3. bölgede cos → −

cot 275° → 4. bölgede → 4. bölgede cot → −

{+, −, −, −}

1988-ÖYS: sin95, cos190, tan210 ifadelerinin işaretlerini bulunuz?

Çözüm:

sin → 95° → 2. bölgede → 2. bölgede sin → +

cos → 190° → 3. bölgede → 3. bölgede cos → −

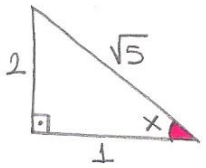
tan → 210° → 3. bölgede → 3. bölgede tan → +

{+, −, +}

Örnek: $0<x<90$, $\cot x = \frac{1}{2}$ ise $\frac{2\sin x+\cos x}{\sin x+2\cos x} = ?$

Çözüm: $0<x<90$ → 1. bölgede,

$\cot x = \frac{1}{2}$ ifadesi için bir dik üçgen çizilerek x açısının trigonometrik değerleri bulunur.



$\sin x = \frac{2}{\sqrt{5}}$ → 1. bölgede → $\sin x = \frac{2}{\sqrt{5}}$

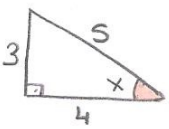
$\cos x = \frac{1}{\sqrt{5}}$ → 1. bölgede → $\cos x = \frac{1}{\sqrt{5}}$

$$\frac{2\sin x+\cos x}{\sin x+2\cos x} = \frac{2.\left(\frac{2}{\sqrt{5}}\right)+\left(\frac{1}{\sqrt{5}}\right)}{\frac{2}{\sqrt{5}}+2.\frac{1}{\sqrt{5}}} = \frac{\frac{5}{\sqrt{5}}}{\frac{4}{\sqrt{5}}} = \frac{5}{4}$$

Örnek: $90<x<180$, $\tan x = -\frac{3}{4}$ ise $\frac{1+2\sin x}{1+2\cos x} = ?$

Çözüm: $90<x<180$ → 2. bölgede

$\tan x = -\frac{3}{4}$ için dik üçgen çizilir.



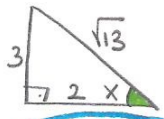
$\sin x = \frac{3}{5}$ → 2. bölgede → $\sin x = \frac{3}{5}$

$\cos x = \frac{4}{5}$ → 2. bölgede → $\cos x = \frac{-4}{5}$

$$\frac{1+2\sin x}{1+2\cos x} = \frac{1+2\left(\frac{3}{5}\right)}{1+2.\left(\frac{-4}{5}\right)} = \frac{1+\frac{6}{5}}{1-\frac{8}{5}} = \frac{\frac{11}{5}}{\frac{3}{5}}$$

Örnek: $\pi < x < \frac{3\pi}{2}$, $\tan x = \frac{3}{2}$, $\cos x - \sin x = ?$

Çözüm: $180 < x < 270 \rightarrow$ 3. bölge



$\tan x = \frac{3}{2}$

$\cos x = \frac{2}{\sqrt{13}}$ → 3. bölgede $\cos x = -\frac{2}{\sqrt{13}}$

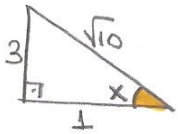
$\sin x = \frac{3}{\sqrt{13}}$ → 3. bölgede $\sin x = -\frac{3}{\sqrt{13}}$

$\cos x - \sin x = \left(-\frac{2}{\sqrt{13}}\right) - \left(-\frac{3}{\sqrt{13}}\right) = \frac{1}{\sqrt{13}}$

Örnek: $\frac{3\pi}{2} < x < 2\pi$, $3\cos x + \sin x = 0$ ise $\sin x$ kaçtır?

Çözüm: $270 < x < 360 \rightarrow$ 4. bölge,

$-3\cos x = \sin x$, $-3 = \frac{\sin x}{\cos x} \Rightarrow \tan x = -3$

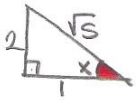


$\tan x = 3$

$\sin x = \frac{3}{\sqrt{10}}$ → 4. bölgede $\sin x = -\frac{3}{\sqrt{10}}$

1982-ÖYS: $\tan x = 2$ ise, $\cos^2 x - \sin x \cdot \cos x = ?$

Çözüm:



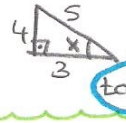
$\tan x = 2$

$\cos^2 x - \sin x \cos x = \left(\frac{1}{\sqrt{5}}\right)^2 - \left(\frac{2}{\sqrt{5}}\right)\cdot\left(\frac{1}{\sqrt{5}}\right)$

$\Rightarrow \frac{1}{5} - \frac{2}{5} = \frac{-1}{5}$

1993-ÖYS: $\frac{3}{\cos x} = \frac{4}{\sin x}$ ise $\cos x$ in pozitif değeri nedir?

Çözüm: $\frac{\sin x}{\cos x} = \frac{4}{3}$ ise $\tan x = \frac{4}{3}$

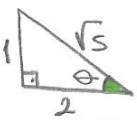


$\tan x = \frac{4}{3}$

$\cos x = \frac{3}{5}$

1974-ÜSS: $0 < \theta < \frac{\pi}{2}$ ve $\tan\theta = \frac{1}{2}$ ise $\cos\theta$ nedir?

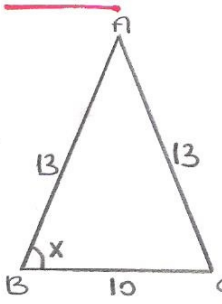
Çözüm:



$\tan\theta = \frac{1}{2}$

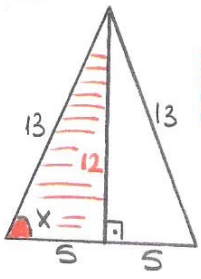
$\cos\theta = \frac{2}{\sqrt{5}}$ → 1. bölgede $\cos\theta = \frac{2}{\sqrt{5}}$

Örnek:



$\sin x = ?$

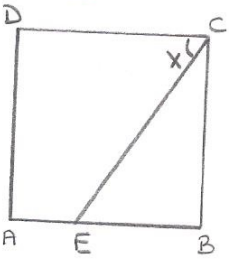
Çözüm:

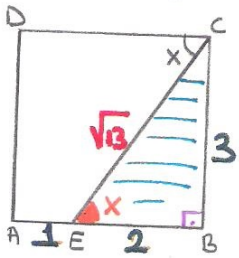


$\sin x = \frac{12}{13}$

Örnek: ABCD kare

Çözüm:



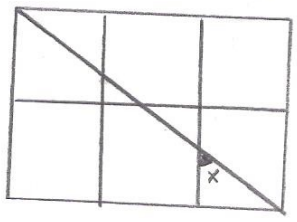


$2|AE| = |EB|$, $\tan x = ?$

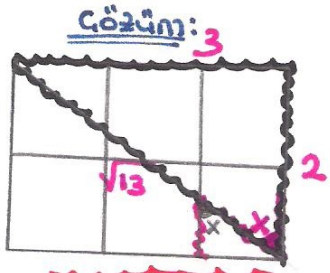
$\tan x = \frac{3}{2}$

Örnek: Birim kareler

Çözüm:

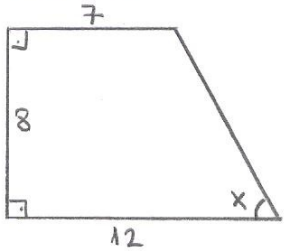


$\cos x = ?$



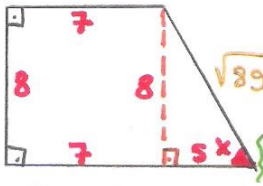
$\cos x = \frac{2}{\sqrt{13}}$

Örnek:



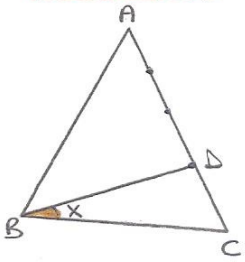
$\tan x = ?$

Çözüm:



$\tan x = \frac{8}{5}$

2009-ÖSS:

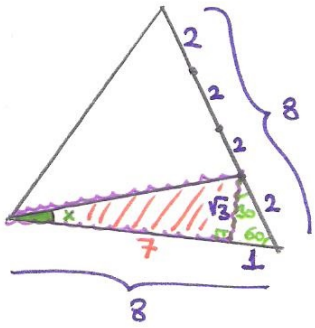


ABC eşkenar üçgen

$|DC| = \frac{1}{4}|AC|$ ise,

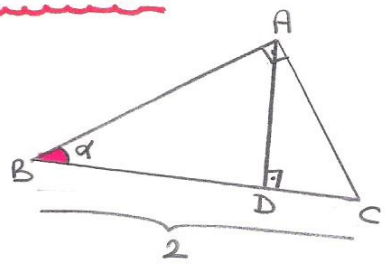
tanx kaçtır?

Çözüm:



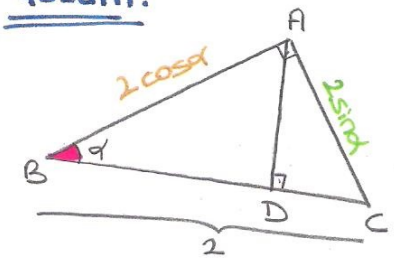
$\tan x = \frac{\sqrt{3}}{7}$

1986-ÖYS:



$|AD| = ?$

Çözüm:



$\triangle ABC$ de, $\cos\alpha = \frac{|AB|}{2} \rightarrow |AB| = 2\cos\alpha$

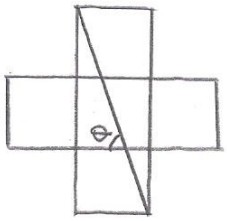
$\sin\alpha = \frac{|AC|}{2} \rightarrow |AC| = 2\sin\alpha$

$A(\triangle ABC) = \frac{2.\sin\alpha.\cancel{2}\cos\alpha}{\cancel{2}} = \frac{\cancel{2}.|AD|}{\cancel{2}}$

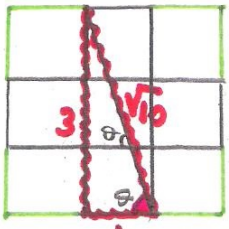
$|AD| = 2\sin\alpha.\cos\alpha$

1983-ÖYS:

Üst tabanı olmayan birim küp

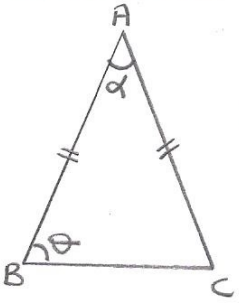


$\tan\theta = ?$



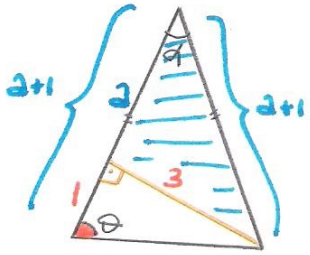
$\tan\theta = 3$

1997-ÖYS:



$\tan\theta = 3$ ise, $\tan\alpha = ?$

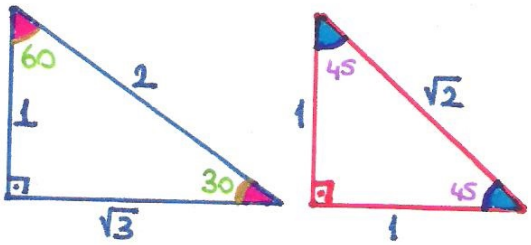
Çözüm:



Pisagordan, $a = 4$

$\tan\alpha = \frac{3}{4}$

Bazı Açıların Trigonometrik Değerleri



$\sin 30 = \frac{1}{2}$, $\tan 30 = \frac{1}{\sqrt{3}}$

$\sin 45 = \frac{\sqrt{2}}{2}$, $\tan 45 = 1$

$\sin 60 = \frac{\sqrt{3}}{2}$, $\tan 60 = \sqrt{3}$

Not: $a + b = 90°$ ise,

$\sin a = \cos b$, $\tan a = \cot b$

Açıların 1. Bölgedeki Değerleri:

* Açı, 1. bölgede ise dokunulmaz.
* Açı, 2. bölgede ise önce trigonometrik ifadenin o bölgedeki işaretine bakılır sonra açı $180°$ ye tamamlanır.
* Açı, 3. bölgede ise önce işaretine bakılır sonra $180°$ çıkarılır.
* Açı, 4. bölgede ise önce işaretine bakılır sonra $360°$ ye tamamlanır.

Not: cosinüs ve cotanjant değerleri için açı $90°$ ye tamamlanarak sinüs veya tanjanta çevrilir.

30 - 45 - 60 → 1. bölge

- $\cos 30 = \sin 60 \rightarrow \cos 30 = \frac{\sqrt{3}}{2}$
- $\cos 45 = \sin 45 \rightarrow \cos 45 = \frac{\sqrt{2}}{2}$
- $\cos 60 = \sin 30 \rightarrow \cos 60 = \frac{\sqrt{3}}{2}$

- $\cot 30 = \tan 60 \rightarrow \cot 30 = \sqrt{3}$
- $\cot 45 = \tan 45 \rightarrow \cot 45 = 1$
- $\cot 60 = \tan 30 \rightarrow \cot 60 = \frac{1}{\sqrt{3}}$

120 - 135 - 150 → 2. bölge

not: 2. bölgede sinüs pozitif diğerleri negatiftir.

* $\sin 120 = +\sin 60 = \frac{\sqrt{3}}{2}$
* $\cos 120 = -\cos 60 = -\sin 30 = -\frac{1}{2}$
* $\tan 135 = -\tan 45 = -1$
* $\cot 150 = -\cot 30 = -\tan 60 = -\sqrt{3}$

Not: Görüldüğü gibi önce açının kaçıncı bölgede olduğuna bakıldı. Sonra trigonometrik ifadenin o bölgedeki işaretine bakıldı. Sonra açı 2. bölgede olduğundan $180°$ ye tamamlanarak trigonometrik ifadesi değişmeden yazıldı.

210 - 225 - 240 → 3. Bölge

* $\sin 225 = -\sin 45 = -\frac{\sqrt{2}}{2}$
* $\cos 210 = -\cos 30 = -\sin 60 = -\frac{\sqrt{3}}{2}$
* $\tan 240 = +\tan 60 = \sqrt{3}$
* $\cot 225 = +\cot 45 = \tan 45 = 1$

Not: Görüldüğü gibi sinüs ve cosinüs 3. bölgede negatif olurken, tanjant ve cotanjant pozitif oldu. Açı 3. bölgede olduğundan $180°$ çıkarıldı.

300 - 315 - 330 → 4. Bölge

* $\sin 300 = -\sin 60 = -\frac{\sqrt{3}}{2}$
* $\cos 315 = +\cos 45 = \sin 45 = \frac{\sqrt{2}}{2}$
* $\tan 330 = -\tan 30 = -\frac{1}{\sqrt{3}}$
* $\cot 315 = -\cot 45 = -\tan 45 = -1$

$(\pi-\alpha)$ → 2. Bölge

* $\sin(\pi-\alpha) = +\sin\alpha$
* $\cos(\pi-\alpha) = -\cos\alpha$
* $\tan(\pi-\alpha) = -\tan\alpha$
* $\cot(\pi-\alpha) = -\cot\alpha$

$(\pi+\alpha)$ → 3. Bölge

* $\sin(\pi+\alpha) = -\sin\alpha$
* $\cos(\pi+\alpha) = -\cos\alpha$
* $\tan(\pi+\alpha) = +\tan\alpha$
* $\cot(\pi+\alpha) = +\cot\alpha$

$(2\pi-\alpha)$ → $(-\alpha)$ → 4. Bölge

* $\sin(-\alpha) = -\sin\alpha$
* $\cos(-\alpha) = +\cos\alpha$
* $\tan(-\alpha) = -\tan\alpha$
* $\cot(-\alpha) = -\cot\alpha$

Uyarı: Açı kaç derece olursa olsun $(-\alpha)$ hep 4. bölgede gibi düşünülür ve bir tek cosinüs pozitif olur.

* $\cos(-\alpha) = \cos\alpha$
* $\cos(-7\alpha) = \cos 7\alpha$
* $\cos(-60) = \cos 60$
* $\cos(-110) = \cos 110$
* $\cos(-240) = \cos 240$

Cosinüs açısı görüldüğü gibi dışarı (+) olarak çıktı.

$\sin(-\alpha) = -\sin\alpha$
$\tan(-7\alpha) = -\tan 7\alpha$
$\cot(-110) = -\cot 110$

Her zaman parantez içindeki açı (+) yapılır. Yani 1. bölgeye taşınır.

Not: $(\alpha - k\pi)$ gibi açılar, esas ölçüleri alınarak işlem yapılır.

* $\sin(x-7\pi) = \sin(x-7\pi+8\pi) = \sin(\pi+x)$
  $\Rightarrow -\sin x$
* $\cos(21\pi-x) = \cos(21\pi-20\pi-x) = \cos(\pi-x)$
  $= -\cos x$

Dikkat: $\left(\frac{\pi}{2}\mp\alpha\right)$ veya $\left(\frac{3\pi}{2}\mp\alpha\right)$ türünden ifadelerle işlem yapmak karışıktır. Dolayısıyla açıyı; 90°ye ekleme 270°ye tamamlama gibi yöntemler kullanmayınız. Ancak soruda bu şekilde karşınıza gelirse,

Önce açının hangi bölgede olduğu bulunur sonra trigonometrik ifadenin o bölgedeki işareti bulunur. Sonra $\frac{\pi}{2}$ veya $\frac{3\pi}{2}$ li açı atılır ve ifade isim değiştirir. $\sin x \leftrightarrow \cos x$, $\tan x \leftrightarrow \cot x$

$\left(\frac{\pi}{2}-\alpha\right)$ → 1. Bölge

$\sin\left(\frac{\pi}{2}-\alpha\right) = +\cos\alpha$

$\tan\left(\frac{\pi}{2}-\alpha\right) = +\cot\alpha$

$\left(\frac{\pi}{2}+\alpha\right)$ → 2. Bölge

$\sin\left(\frac{\pi}{2}+\alpha\right) = +\cos\alpha$, $\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha$

$\cot\left(\frac{\pi}{2}+\alpha\right) = -\tan\alpha$

$\left(\frac{3\pi}{2}-\alpha\right)$ → 3. Bölge

$\cos\left(\frac{3\pi}{2}-\alpha\right) = -\sin\alpha$

$\cot\left(\frac{3\pi}{2}-\alpha\right) = +\tan\alpha$

$\left(\frac{3\pi}{2}+\alpha\right)$ → 4. Bölge

$\sin\left(\frac{3\pi}{2}+\alpha\right) = -\cos\alpha$

$\cos\left(\frac{3\pi}{2}+\alpha\right) = +\sin\alpha$

$\tan\left(\frac{3\pi}{2}+\alpha\right) = -\cot\alpha$

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Başarıya açılan pencereniz...

Not: $a+b=90^\circ$ ise

$\sin a = \cos b$ ve $\tan a = \cot b$

✓ $7x = \frac{\pi}{2}$ ise,

- $\sin 6x = \cos x$
- $\sin 4x = \cos 3x$
- $\tan 5x = \cot 2x$

✓ $11x = \frac{\pi}{4} \rightarrow 22x = \frac{\pi}{2}$ ise,

- $\cos 20x = \sin 2x$
- $\tan 17x = \cot 5x$

Not: $a+b=180^\circ$ ise

$\sin a = \sin b$

$\cos a = -\cos b$
$\tan a = -\tan b$
$\cot a = -\cot b$

✓ $7x = \pi$ ise

- $\sin 6x = \sin x$
- $\cos 5x = -\cos 2x$
- $\tan 4x = -\tan 3x$
- $\cot 5x = -\cot 2x$

✓ $3x = \frac{\pi}{7} \rightarrow 21x = \pi$ ise

- $\sin 19x = \sin 2x$
- $\cos 15x = -\cos 6x$
- $\tan 12x = -\tan 9x$
- $\cot 11x = -\cot 10x$

Not: $a+b=360^\circ$ ise

$\cos a = \cos b$

$\sin a = -\sin b$
$\tan a = -\tan b$
$\cot a = -\cot b$

✓ $15x = 2\pi$ ise

- $\cos 13x = \cos 2x$
- $\sin 10x = -\sin 5x$
- $\cot 8x = -\cot 7x$
- $\tan 12x = -\tan 3x$

✓ $11x = \frac{2\pi}{3} \rightarrow 33x = 2\pi$ ise

- $\cos 30x = \cos 3x$
- $\sin 27x = -\sin 6x$
- $\tan 18x = -\tan 15x$
- $\cot 20x = -\cot 13x$

* $a+b=30^\circ$ ise $\sin(4a+3b) = ?$

* $\sin(\underbrace{3a+3b}_{90}+a) = \sin(90+a) = \cos a$

* $a+b=90^\circ$ ise

$\sin^2 a + \sin^2 b = 1$, $\cos^2 a + \cos^2 b = 1$

$\tan a . \tan b = 1$, $\cot a . \cot b = 1$

* $\sin^2 1 + \sin^2 2 + \_\_ + \sin^2 45 + \_\_\_\_ + \sin^2 90 = ?$

Çözüm: $\sin^2 1 + \sin^2 89 = 1$ ... $\underbrace{\sin^2 44 + \sin^2 46 = 1}_{44}$

$\sin^2 45 = \frac{1}{2}$

$\sin^2 90 = 1$

$= 44 + 1 + \frac{1}{2} = \frac{91}{2}$

* $\tan 1 . \tan 2 . \_\_ . \tan 89 = ?$

Çözüm: $\tan 1 . \tan 89 = 1$

$\tan 44 . \tan 46 = 1$

$\underbrace{1 . 1 . \_\_\_ . 1}_{45 \text{ tane}} = 1$

$\tan 45 = 1$

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Başarıya açılan pencereniz...

Ek Sayfa

cyhnyv1

1986-ÖYS: Hangisi $\sin 40$ a eşittir?

- $\sin 220 \rightarrow -\sin 40$
- $\cos 130 \rightarrow -\cos 50$
- $\sin 50 \rightarrow \sin 50$
- $\sin(-40) \rightarrow -\sin 40$
- $\cos(-50) \rightarrow \cos 50 = \sin 40$

1974-ÜSS: $\sin 210$ un değeri kaçtır?

Çözüm: $\sin 210 = -\sin 30 = \frac{-1}{2}$

2008-ÖSS: $\cos\left(\frac{\pi}{2}+x\right) = \sin\left(\frac{\pi}{2}-x\right)$ ise $\tan x = ?$

Çözüm: $-\sin x = \cos x \rightarrow \frac{-\sin x}{\cos x} = \frac{\cos x}{\cos x}$

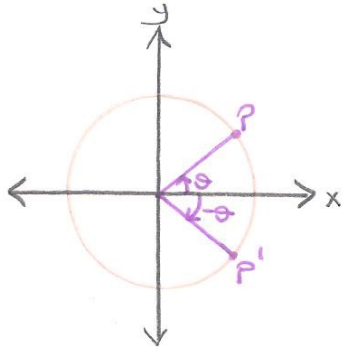
$\Rightarrow -\tan x = 1$ ise $\tan x = -1$

1984-ÖYS: Hangisi $\sin\left(\frac{\pi}{2}-a\right)$ ya eşit değildir?

Not: $\sin\left(\frac{\pi}{2}-a\right) = \cos a$ olmalı.

- $\sin\left(\frac{\pi}{2}+a\right) = \cos a$
- $\sin(-a) = -\sin a$
- $\cos(-a) = +\cos a$
- $\cos a = \cos a$
- $\cos(2\pi - a) = \cos a$

2006-ÖSS:



P' noktası, aşağıdakilerden hangisi ile ifade edilemez?

- $(\cos(-\theta), \sin(-\theta)) \rightarrow (\cos\theta, -\sin\theta)$
- $(\cos(-\theta), \sin\theta) \rightarrow (\cos\theta, \sin\theta)$
- $(\cos\theta, -\sin\theta)$
- $(\cos\theta, \sin(2\pi-\theta)) \rightarrow (\cos\theta, -\sin\theta)$
- $(\cos(2\pi-\theta), -\sin\theta) \rightarrow (\cos\theta, -\sin\theta)$

Örnek: $\frac{\cos 316 . \sin 43}{\sin 46 . \cos 1753} = ?$

Çözüm: $1753°$ nin esas ölçüsü : $313°$

- $\cos 316 \rightarrow \cos 44 = \sin 46$
- $\cos 1753 \rightarrow \cos 313 = \cos 47 = \sin 43$

$$\frac{\cos 316 . \sin 43}{\sin 46 . \cos 1753} = \frac{\sin 46 . \sin 43}{\sin 46 . \sin 43} = 1$$

Örnek: $\frac{\cos(-120) . \sin 330}{\cos 150 . \sin 300} = ?$

Çözüm:

- $\cos(-120) \rightarrow -\cos 60 = -\sin 30 = -\frac{1}{2}$
- $\sin(330) \rightarrow -\sin 30 = -\frac{1}{2}$
- $\cos 150 \rightarrow -\cos 30 = -\sin 60 = -\frac{\sqrt{3}}{2}$
- $\sin 300 \rightarrow -\sin 60 = -\frac{\sqrt{3}}{2}$

$$\frac{\cos(-120) . \sin 330}{\cos 150 . \sin 300} = \frac{-\frac{1}{2} . -\frac{1}{2}}{-\frac{\sqrt{3}}{2} . -\frac{\sqrt{3}}{2}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Örnek: $\pi = 7a$ ise $\frac{\sin 3a . \cos 5a}{\sin 4a . \cos 2a} = ?$

- $\pi = 7a$ ise $\sin 3a = \sin 4a$,
- $\pi = 7a$ ise $\cos 5a = -\cos 2a$

$$\frac{\sin 3a . \cos 5a}{\sin 4a . \cos 2a} = -1$$

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Başarıya açılan pencereniz...

Trigonometrik ifadelerin Karşılaştırılması

* Verilen trigonometrik ifadelerin I. Bölgeye karşılık gelen değeri bulunur.

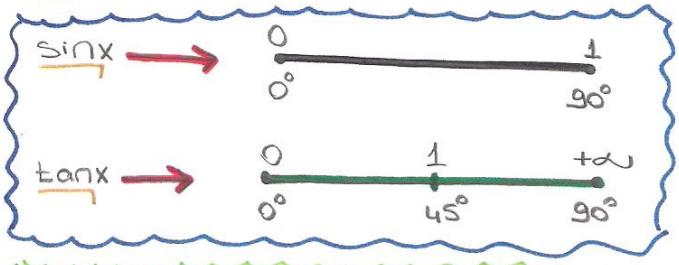
* Cosinüs varsa sinüse, cotanjant varsa tanjanta çevrilir.

* 1. Bölgede $\alpha > \beta$ olsun. O halde, $\sin\alpha > \sin\beta$, $\tan\alpha > \tan\beta$

* 1. bölgedeki bir açı için her zaman $\tan\alpha > \sin\alpha$

Çünkü, $\tan\alpha = \frac{\text{Karşı}}{\text{Komşu}}$, $\sin\alpha = \frac{\text{Karşı}}{\text{Hip.}}$

Hipotenüs > Komşu → $\tan\alpha > \sin\alpha$



Yani, $a > b$, $a = b$, $a > 45°$ ise, $\tan a > \sin b$

1985-ÖYS: $a = \sin 5$, $b = \sin 85$, $c = \sin 105$ ise a, b, c yi sıralayınız?

Çözüm:

- $a = \sin 5$
- $b = \sin 85$
- $c = \sin 105 = \sin 75$

$\sin 85 > \sin 75 > \sin 5$ → $b > c > a$

Örnek: $a = \sin 120$, $b = \cos 50$, $c = \tan 50$ ise a, b, c yi sıralayınız?

Çözüm:

* $\sin 120 = \sin 60$
* $\cos 50 = \sin 40$
* $\tan 50$

$\tan 50 > \sin 60 > \sin 40$ → $c > a > b$

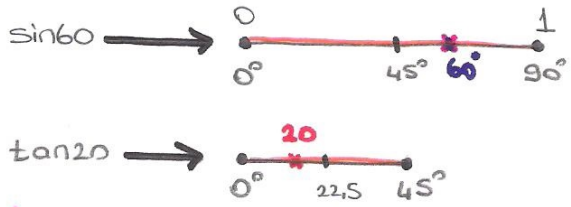
Örnek: $a = \cos 330$, $b = \sin 210$, $c = \tan 70$ $d = \cot 250$ ise a, b, c, d yi sıralayın?

Çözüm:

- $a = \cos 330 = \cos 30 = \sin 60$
- $b = \sin 210 = -\sin 30$
- $c = \tan 70$
- $d = \cot 250 = \cot 70 = \tan 20$

en büyük → c , en küçük → b

peki sin60 mı tan20 mi daha büyük?

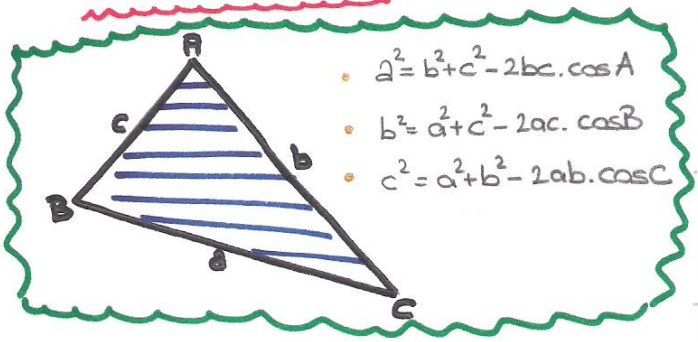


✓ Sinüs için [0,1] aralığı yaklaşık 90 parçaya bölünmüş ve 60° orta değerin sağında yani 1'e yakın olan taraftadır.

✗ tanjant için [0,1] aralığı yaklaşık 45 parçaya bölünmüş ve 20° orta değerin solunda yer almaktadır. Yani 0'a yakın olan taraftadır. $\sin 60 > \tan 20$

O halde, $c > a > d > b$

Cosinüs Teoremi



- $a^2 = b^2 + c^2 - 2bc.\cos A$
- $b^2 = a^2 + c^2 - 2ac.\cos B$
- $c^2 = a^2 + b^2 - 2ab.\cos C$

1971-üss: $a^2 = b^2 + c^2 + bc$ ise $\hat{A} = ?$

Çözüm:

$a^2 = b^2 + c^2 + bc$ → Soruda verilen

$-/\ a^2 = b^2 + c^2 - 2bc.\cos A$ → cosinüs teoremi

$0 = 2bc.\cos A + bc$, $-bc = 2bc.\cos A$

$-\frac{1}{2} = \cos A \rightarrow A = 120^\circ$

Örnek: Bir üçgenin kenarları a,b,c ve $a^2 = b^2 + c^2 - \sqrt{2}bc$ ise $m(\hat{A})$ kaçtır?

Çözüm:

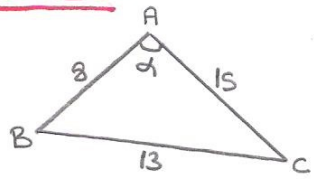
$a^2 = b^2 + c^2 - \sqrt{2}bc$ → Soruda verilen

$-/\ a^2 = b^2 + c^2 - 2bc.\cos A$ → cosinüs teoremi

$0 = 2bc.\cos A - \sqrt{2}.bc$, $\sqrt{2}bc = 2bc.\cos A$

$\frac{\sqrt{2}}{2} = \cos A \rightarrow A = 45^\circ$

Örnek:



α kaç derecedir?

Çözüm:

$13^2 = 8^2 + 15^2 - 2.8.15.\cos\alpha$

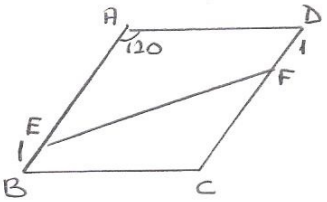
$169 = 64 + 225 - 240.\cos\alpha$

$240.\cos\alpha = 289 - 169$

$240\cos\alpha = 120$, $\cos\alpha = \frac{1}{2}$

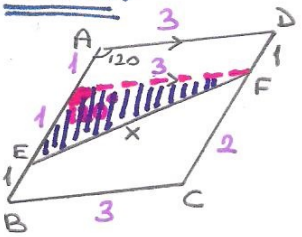
$\alpha = 60^\circ$

1980-üss:



ABCD eşkenar dörtgen $|AB| = 3$ ise $|EF| = ?$

Çözüm:



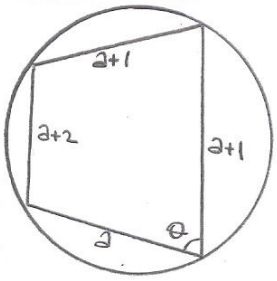
$x^2 = 1^2 + 3^2 - 2.1.3.\cos 120$

$x^2 = 1 + 9 - 6.(-\frac{1}{2})$

$x^2 = 10 + 3$

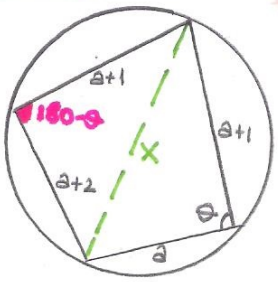
$x^2 = 13$, $x = \sqrt{13}$

1975-üss:



$\cos\theta$ kaçtır?

Çözüm:



* Kirişler dörtgeninden θ nın karşısı 180-θ dır.
* İki tane cosinüs teoremi uygulanıp taraf tarafa toplanır.

$-/\ x^2 = a^2 + (a+1)^2 - 2.a.(a+1).\cos\theta$ ($\cos(180-\theta) = -\cos\theta$)

$x^2 = (a+1)^2 + (a+2)^2 - 2(a+1).(a+2).\underbrace{\cos(180-\theta)}_{-\cos\theta}$

$0 = -a^2 + (a+2)^2 + 2.a(a+1).\cos\theta - 2.(a+1).(a+2).(-\cos\theta)$

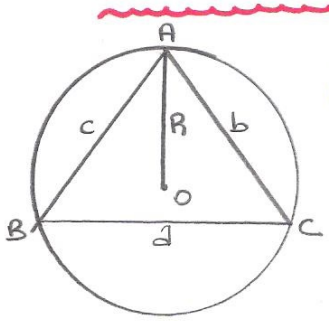
$0 = -a^2 + a^2 + 4a + 4 + 2.(a+1).\cos\theta.(a+a+2)$

$0 = 4(a+1) + 2.(a+1).\underbrace{(2a+2)}_{2(a+1)}.\cos\theta$

$-4(a+1) = 4(a+1)(a+1).\cos\theta$

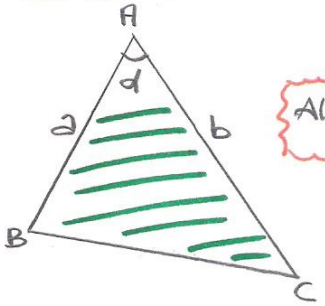
$-1 = (a+1).\cos\theta$, $\cos\theta = \frac{-1}{a+1}$

Sinüs Teoremi



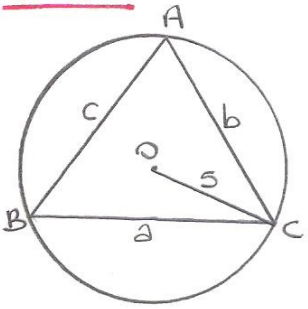
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

Sinüs ile Alan Teoremi :



$$A(\triangle ABC) = \frac{1}{2}.a.b.\sin\alpha$$

Örnek:



$\sin\hat{A}+\sin\hat{B}+\sin\hat{C}=\frac{3}{2}$ ise $C(\triangle ABC)$ kaçtır?

Çözüm: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R = 2.5 = 10$

$a = 10.\sin\hat{A} \rightarrow \sin\hat{A} = \frac{a}{10}$

$b = 10.\sin\hat{B} \rightarrow \sin\hat{B} = \frac{b}{10}$

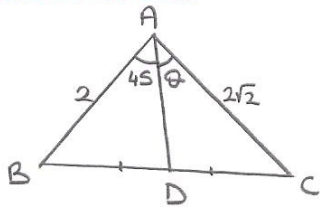
$c = 10.\sin\hat{C} \rightarrow \sin\hat{C} = \frac{c}{10}$

$\sin\hat{A}+\sin\hat{B}+\sin\hat{C}=\frac{3}{2}$ ise,

$\frac{a}{10}+\frac{b}{10}+\frac{c}{10}=\frac{3}{2}$, $\frac{a+b+c}{10}=\frac{3}{2}$

$a+b+c=\frac{30}{2}=15$, $C(\triangle ABC)=15$

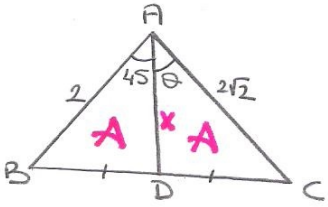
1991 - ÖYS:



$\sin\theta = ?$

Çözüm:

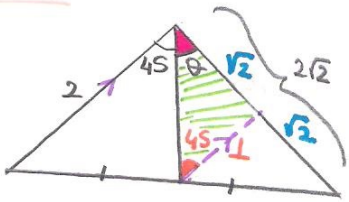
1. Yol: $|BD| = |DC|$ ise, $A(\triangle ABD) = A(\triangle ADC)$



$\frac{1}{2}.2.x.\sin 45 = \frac{1}{2}.x.2\sqrt{2}.\sin\theta$

$\frac{\sqrt{2}}{2} = \sqrt{2}.\sin\theta$, $\sin\theta = \frac{1}{2}$

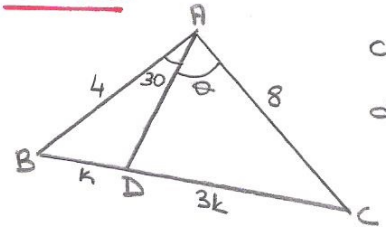
2. Yol:



Taralı üçgende,

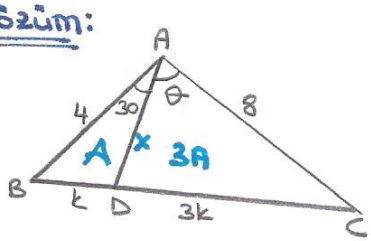
$\frac{1}{\sin\theta} = \frac{\sqrt{2}}{\sin 45}$, $\sqrt{2}.\sin\theta = \frac{\sqrt{2}}{2}$, $\sin\theta = \frac{1}{2}$

Örnek:



$\cos\theta$ nın pozitif değeri kaçtır?

Çözüm:

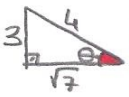


$\frac{1}{2}.4.x.\sin 30 = A$

$\frac{1}{2}.x.8.\sin\theta = 3A$

$3\sin 30 = 2\sin\theta$

$3.\frac{1}{2} = 2\sin\theta$, $\sin\theta = \frac{3}{4}$



$\cos\theta = \frac{\sqrt{7}}{4}$

$\sin\theta = \frac{3}{4}$

Bilinmesi Gereken Trigonometri Formülleri :

sin(a+b)= sina.cosb+sinb.cosa

sin(a-b)= sina.cosb−sinb.cosa

cos(a+b)= cosa.cosb − sina.sinb

cos(a-b)= cosa.cosb + sina.sinb

sin2a= 2.sina.cosa

cos2a= $\cos^2a-\sin^2a$

$\cos^2a+\sin^2a=1$

$\cos^2a=1-\sin^2a$ $\quad$ $\sin^2a=1-\cos^2a$

cosa.cosb= $\frac{\cos(a+b)+\cos(a-b)}{2}$

not: cosa.cosb bilinirse, "sina.cosb", "sinb.cosa" ve "sina.sinb" formüllerini ezberlemek gereksizdir.

sina+sinb= $2.\sin\left(\frac{a+b}{2}\right).\cos\left(\frac{a-b}{2}\right)$

sina−sinb= $2.\sin\left(\frac{a-b}{2}\right).\cos\left(\frac{a+b}{2}\right)$

not: sina+sinb ve sina−sinb bilinirse "cosa+cosb", "cosa−cosb" formüllerini ezberlemek gereksizdir.

tan(a+b)= $\frac{\tan a+\tan b}{1-\tan a.\tan b}$, **tan(a-b)=** $\frac{\tan a-\tan b}{1+\tan a.\tan b}$

$\tan2a=\frac{2\tan a}{1-\tan^2a}$

cot(a+b) → $\frac{1}{\tan(a+b)}$

cot(a-b) → $\frac{1}{\tan(a-b)}$

cot2a= $\frac{1}{\tan2a}$

sin(a+b)= sina.cosb+sinb.cosa

Örnek: sin75 in değeri nedir?

Çözüm: $\sin(45+30)=\sin45.\cos30+\sin30.\cos45$

$=\frac{\sqrt2}{2}.\frac{\sqrt3}{2}+\frac{1}{2}.\frac{\sqrt2}{2}=\frac{\sqrt6+\sqrt2}{4}$

Örnek: $\frac{\sin10.\cos30+\sin30.\cos10}{\sin90.\cos50+\cos90.\sin50}=?$

Çözüm: $\frac{\sin(10+30)}{\sin(90+50)}=\frac{\sin40}{\sin140}=\frac{\sin40}{\sin40}$

$\Rightarrow 1$

sin(a-b)= sina.cosb−sinb.cosa

Örnek: sin72.cos27 − sin27.cos72 = ?

Çözüm: $\sin(72-27)=\sin45=\frac{\sqrt2}{2}$

cos(a+b)= cosa.cosb−sina.sinb

Örnek: cos105 in değeri nedir?

Çözüm: $\cos(60+45)=\cos60.\cos45-\sin60.\sin45$

$\frac{1}{2}.\frac{\sqrt2}{2}-\frac{\sqrt3}{2}.\frac{\sqrt2}{2}=\frac{\sqrt2-\sqrt6}{4}$

Örnek: cos(3x+y) ifadesini açınız?

Çözüm: cos3x.cosy − sin3x.siny

cos(a-b)= cosa.cosb+sina.sinb

Örnek: cos20.cos50+sin20.sin50= ?

Çözüm: $\cos(20-50)=\cos(-30)$

$=\cos30=\sin60=\frac{\sqrt3}{2}$

$\sin 2a = 2\sin a.\cos a$

Örnek: $\sin x.\cos x$ ifadesinin en sade halini bulunuz?

Çözüm: $\sin x.\cos x = \frac{2\sin x\cos x}{2} = \frac{\sin 2x}{2}$

Örnek: $\sin 15.\cos 15$ değeri kaçtır?

Çözüm: $\sin 15.\cos 15 = \frac{2.\sin 15.\cos 15}{2} = \frac{\sin 30}{2} = \frac{1}{4}$

Örnek: $\cos 20.\cos 40.\cos 80 = ?$

Çözüm:

$$\frac{2.\sin 20.\cos 20.\cos 40.\cos 80}{2.\sin 20} \quad (\sin 40)$$

$$= \frac{2.\sin 40\cos 40\cos 80}{2.2.\sin 20} \quad (\sin 80)$$

$$= \frac{2.\sin 80.\cos 80}{2.4.\sin 20} \quad (\sin 160)$$

$$\Rightarrow \frac{\sin 160}{8\sin 20} = \frac{\sin 20}{8\sin 20} = \frac{1}{8}$$

Örnek: $\cos 20.\cos 40\cos 60.\cos 80 = ?$

Çözüm: $\underbrace{\cos 20\cos 40\cos 80}_{1/8}.\underbrace{\cos 60}_{1/2} = \frac{1}{16}$

Örnek: $\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{4\pi}{7} = ?$

Çözüm:

$$\frac{2.\sin\frac{\pi}{7}\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{4\pi}{7}}{2.\sin\frac{\pi}{7}} \quad (\sin\frac{4\pi}{7})$$

$$= \frac{2.\sin\frac{2\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{4\pi}{7}}{2.2.\sin\frac{\pi}{7}} \quad (\sin\frac{8\pi}{7})$$

$$\Rightarrow \frac{2.\sin\frac{4\pi}{7}.\cos\frac{4\pi}{7}}{2.4.\sin\frac{\pi}{7}} = \frac{\sin\frac{8\pi}{7}}{8.\sin\frac{\pi}{7}} = \frac{\sin\left(\pi+\frac{\pi}{7}\right)}{8.\sin\frac{\pi}{7}}$$

$$= \frac{-\sin\frac{\pi}{7}}{8.\sin\frac{\pi}{7}} = \frac{-1}{8}$$

2011-LYS: $0 < x < \frac{\pi}{2}$ olmak üzere,

$\cot x - 3\tan x = \frac{1}{\sin 2x}$ ise $\sin^2 x$ kaçtır?

Çözüm: $\frac{\cos x}{\sin x} - \frac{3.\sin x}{\cos x} = \frac{1}{2\sin x.\cos x}$ (cosx) (sinx)

$$\frac{\cos^2 x - 3\sin^2 x}{\sin x.\cos x} = \frac{1}{2.\sin x.\cos x}$$

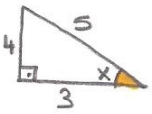
$$(1-\sin^2 x) - 3\sin^2 x = \frac{1}{2}, \quad 1 - 4\sin^2 x = \frac{1}{2}$$

$$1 - \frac{1}{2} = 4\sin^2 x, \quad 4\sin^2 x = \frac{1}{2}, \quad \sin^2 x = \frac{1}{8}$$

1983-ÖYS: $0 < x < \frac{\pi}{2}$, $\tan x = \frac{4}{3}$ ise

$\frac{\sin^3 x - \cos^3 x}{1 + \frac{1}{2}.\sin 2x}$ ifadesinin değeri kaçtır?

Çözüm:



$\tan x = \frac{4}{3}$

$$\frac{\left(\frac{4}{5}\right)^3 - \left(\frac{3}{5}\right)^3}{1 + \frac{1}{2}.2.\sin x.\cos x} \quad \left(\frac{4}{5}.\frac{3}{5}\right)$$

$$= \frac{\frac{64-27}{125}}{1+\frac{12}{25}} = \frac{\frac{37}{125}}{\frac{37}{25}} = \frac{1}{5}$$

2010-LYS: $\frac{\tan 60}{\sin 20} - \frac{1}{\cos 20} = ?$

Çözüm:

$$\frac{\frac{\sin 60}{\cos 60}}{\sin 20} - \frac{1}{\cos 20} = \frac{\sin 60}{\sin 20.\cos 60} - \frac{1}{\cos 20} \quad (\cos 20)\ (\sin 20.\cos 60)$$

$$= \frac{\sin 60.\cos 20 - \sin 20.\cos 60}{\sin 20.\cos 20.\cos 60} = \frac{\sin(60-20)}{\sin 20.\cos 20.\cos 60}$$

$$= \frac{\sin 40}{\sin 20.\cos 20.\cos 60} = \frac{2.\sin 20.\cos 20}{\sin 20.\cos 20.\frac{1}{2}} = 4$$

2008-ÖSS: $\sin 2x = a$ ise $(\sin x + \cos x)^2 = ?$

Çözüm: $\underbrace{\sin^2 x + \cos^2 x}_{1} + \underbrace{2\sin x \cos x}_{\sin 2x} = 1 + a$

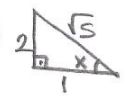
2007-ÖSS: $\left(\sin\frac{\pi}{12} + \cos\frac{\pi}{12}\right)^2 = ?$

Çözüm: $(\sin 15 + \cos 15)^2 = \underbrace{\sin^2 15 + \cos^2 15}_{1} + \underbrace{2\sin 15 \cos 15}_{+\ \sin 30}$

$= 1 + \frac{1}{2} = \boxed{\frac{3}{2}}$

1972-ÜSS: $\tan x = 2$ ise $\sin 2x$ nedir?

Çözüm:



$\tan x = 2$

$\sin 2x = 2\sin x \cdot \cos x$

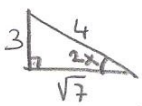
$\sin 2x = 2 \cdot \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} = \boxed{\frac{4}{5}}$

1994-ÖYS: $\cos x - \sin x = \frac{1}{2}$ ise $\cos 2x = ?$

Çözüm: $(\cos x - \sin x)^2 = \left(\frac{1}{2}\right)^2$

$\Rightarrow \underbrace{\cos^2 x + \sin^2 x}_{1} - \underbrace{2\sin x \cos x}_{\sin 2x} = \frac{1}{4}$

$\Rightarrow 1 - \frac{1}{4} = \sin 2x \ , \ \sin 2x = \frac{3}{4}$



$\sin 2x = \frac{3}{4}$

$\boxed{\cos 2x = \frac{\sqrt{7}}{4}}$

1967-ÜSS: $\sin x - \cos x = \frac{1}{2}$ ise $\sin 2x = ?$

Çözüm: $(\sin x - \cos x)^2 = \left(\frac{1}{2}\right)^2$

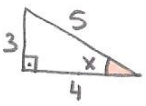
$\underbrace{\sin^2 x + \cos^2 x}_{1} - \underbrace{2\sin x \cos x}_{\sin 2x} = \frac{1}{4}$

$1 - \frac{1}{4} = \sin 2x \ , \ \boxed{\sin 2x = \frac{3}{4}}$

$\boxed{\cos 2a = \cos^2 a - \sin^2 a}$

2011-LYS: $\cos x = \frac{-4}{5}$ ise $\cos 2x = ?$

Çözüm:



$\cos x = \frac{-4}{5}$

$\cos 2x = \cos^2 x - \sin^2 x = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$

1989-ÖYS: $\cos 36 = \frac{\sqrt{5}+1}{4}$ ise $\cos 72 = ?$

Çözüm: $\cos 72 = \cos(36+36) = \cos^2 36 - \sin^2 36$

$= \cos^2 36 - (1 - \cos^2 36) = 2\cos^2 36 - 1$

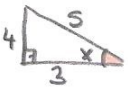
$= 2 \cdot \left(\frac{\sqrt{5}+1}{4}\right)^2 - 1 = \frac{2}{16} \cdot \left(\frac{5+1+2\sqrt{5}}{8}\right) - \frac{1}{(8)}$

$= \frac{6+2\sqrt{5}-8}{8} = \frac{2\sqrt{5}-2}{8} = \boxed{\frac{\sqrt{5}-1}{4}}$

2010-LYS: $3\sin x - 4\cos x = 0$, $|\cos 2x| = ?$

Çözüm: $3\sin x = 4\cos x$, $\frac{\sin x}{\cos x} = \frac{4}{3}$

$\tan x = \frac{4}{3}$



$|\cos 2x| = |\cos^2 x - \sin^2 x| = \left|\left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2\right|$

$= \left|\frac{9}{25} - \frac{16}{25}\right| = \left|\frac{-7}{25}\right| = \boxed{\frac{7}{25}}$

2006-ÖSS: $\frac{\sin 2a}{1 - \cos 2a}$ sonucu nedir?

Çözüm: $\frac{2\sin a \cos a}{1 - (\cos^2 a - \sin^2 a)} = \frac{2\sin a \cos a}{\underbrace{1 - \cos^2 a}_{\sin^2 a} + \sin^2 a}$

$= \frac{2\sin a \cos a}{2\sin^2 a} = \frac{2\sin a \cos a}{2\sin a \cdot \sin a} = \boxed{\cot a}$

2007-ÖSS: $\dfrac{\cos 2a}{1-\tan^2 a} = ?$

Çözüm: $\dfrac{\cos^2 a - \sin^2 a}{1 - \frac{\sin^2 a}{\cos^2 a}} = \dfrac{\cos^2 a - \sin^2 a}{\frac{\cos^2 a - \sin^2 a}{\cos^2 a}}$ ($\frac{1}{\cos^2 a}$)

$= \dfrac{\cancel{\cos^2 a - \sin^2 a}}{1} \cdot \dfrac{\cos^2 a}{\cancel{\cos^2 a - \sin^2 a}} = \boxed{\cos^2 a}$

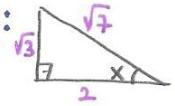
1981-ÖYS: $\tan x = \dfrac{\sin 2y}{1-\cos 2y}$ ise $x+y = ?$

Çözüm: $\dfrac{\sin 2y}{1-\cos 2y} = \dfrac{\cancel{2}\sin y \cdot \cos y}{\cancel{2}\sin y \cdot \sin y} = \cot y$

$\tan x = \cot y$ ise $\boxed{x+y = 90^\circ}$

1973-ÜSS: $\tan x = \dfrac{\sqrt{3}}{2}$ ise $\cos 2x = ?$

Çözüm:

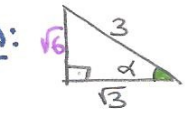


$\tan x = \frac{\sqrt{3}}{2}$

$\cos 2x = \cos^2 x - \sin^2 x = \left(\frac{2}{\sqrt{7}}\right)^2 - \left(\frac{\sqrt{3}}{\sqrt{7}}\right)^2 = \boxed{\frac{1}{7}}$

1974-ÜSS: $\cos\alpha = \dfrac{\sqrt{3}}{3}$ ise $0 < \alpha < \dfrac{\pi}{2}$ için $\cos 2\alpha$ kaçtır?

Çözüm:



$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \left(\frac{\sqrt{3}}{3}\right)^2 - \left(\frac{\sqrt{6}}{3}\right)^2 = \boxed{\frac{-1}{3}}$

Örnek: $\cos 33 = a$ ise $\cos 66 = ?$

Çözüm: $\cos 66 = \cos^2 33 - \underbrace{\sin^2 33}_{1-\cos^2 33}$

$\cos 66 = 2\cos^2 33 - 1 = \boxed{2a^2 - 1}$

Örnek: $\sin 55 = m$ ise $\sin 20$ kaçtır?

Çözüm: $\sin 55 = \cos 35 = m$

$\sin 20 = \cos 70 = ?$

$\cos 70 = \cos^2 35 - \underbrace{\sin^2 35}_{1-\cos^2 35} = \cos^2 35 - (1 - \cos^2 35)$

$= 2\cos^2 35 - 1 = \boxed{2m^2 - 1}$

Örnek: $\sqrt{\dfrac{1-\cos 66}{2}} = ?$

Çözüm: $\sqrt{\dfrac{1-(\cos^2 33 - \sin^2 33)}{2}}$

$= \sqrt{\dfrac{1-\cos^2 33 + \sin^2 33}{2}} = \sqrt{\dfrac{\sin^2 33 + \sin^2 33}{2}}$

$= \sqrt{\dfrac{\cancel{2}\sin^2 33}{\cancel{2}}} = \sqrt{\sin^2 33} = \boxed{\sin 33}$

Örnek: $\cos 64 = a$ ise $\sin 58 = ?$

Çözüm: $\sin 58 = \cos 32 = ?$

$\cos 2a = \underbrace{\cos^2 a - \sin^2 a}_{1-\cos^2 a} = 2\cos^2 a - 1$

$\underbrace{\cos 64}_{a} = 2\cos^2 32 - 1$, $a+1 = 2\cos^2 32$

$\cos^2 32 = \dfrac{a+1}{2}$, $\cos 32 = \sqrt{\dfrac{a+1}{2}}$

$\boxed{\sin 58 = \sqrt{\dfrac{a+1}{2}}}$

$\cos a.\cos b = \frac{\cos(a+b)+\cos(a-b)}{2}$

Örnek: $\cos 15.\cos 75 = ?$

Çözüm: $\frac{\cos(15+75)+\cos(15-75)}{2}$

$\Rightarrow \frac{\cos 90+\cos(-60)}{2} = \frac{\cos(-60)}{2} = \frac{\cos 60}{2} = \frac{1}{4}$

Örnek: $\cos 75.\sin 135 = ?$

Çözüm: $\cos 75.\sin 45 = \cos 75.\cos 45$

$\Rightarrow \frac{\cos(75+45)+\cos(75-45)}{2} = \frac{\cos 120+\cos 30}{2}$

$\Rightarrow \frac{-\cos 60+\cos 30}{2} = \frac{-\frac{1}{2}+\frac{\sqrt{3}}{2}}{2} = \frac{\sqrt{3}-1}{4}$

Örnek: $\sin 105.\sin 165 = ?$

Çözüm: $\sin 75.\sin 15 = \cos 15.\cos 75$

$\Rightarrow \frac{\cos(15+75)+\cos(15-75)}{2} = \frac{\cos 90+\cos(-60)}{2}$

$= \frac{\cos 60}{2} = \frac{\frac{1}{2}}{2} = \frac{1}{2}.\frac{1}{2} = \frac{1}{4}$

Örnek: $\cos 3x.\cos x = ?$

Çözüm: $\frac{\cos(3x+x)+\cos(3x-x)}{2} = \frac{\cos 4x+\cos 2x}{2}$

Örnek: $\cos 4x.\sin 2x = ?$

Çözüm: $\cos 4x.\cos(90-2x)$

$\Rightarrow \frac{\cos(4x+90-2x)+\cos(4x-(90-2x))}{2}$

$6x-90 \Rightarrow$
$6x-90+360$
$\Rightarrow 270+6x$

$\Rightarrow \frac{\cos(90+2x)+\cos(6x-90)}{2}$

$\frac{\cos(90+2x)+\cos(270+6x)}{2} = \frac{-\sin 2x+\sin 6x}{2}$

Örnek: $\sin 3x.\sin 5x = ?$

Çözüm: $\cos(90-3x).\cos(90-5x)$

$\Rightarrow \frac{\cos(90-3x+90-5x)+\cos(90-3x-(90-5x))}{2}$

$\Rightarrow \frac{\cos(180-8x)+\cos(2x)}{2} = \frac{-\cos 8x+\cos 2x}{2}$

Sonuç: Sadece $\cos a.\cos b$ formülü ile "$\sin a.\cos b$", "$\sin b.\cos a$", "$\sin a.\sin b$" formüllerine gerek kalmadan sonuç bulunabilir.

$\sin a+\sin b = 2.\sin\left(\frac{a+b}{2}\right).\cos\left(\frac{a-b}{2}\right)$

Örnek: $\sin 15+\cos 15 = ?$

Çözüm: $\sin 15+\sin 75 = 2.\sin\left(\frac{15+75}{2}\right).\cos\left(\frac{15-75}{2}\right)$

$\Rightarrow 2.\sin 45.\cos(-30) = 2.\sin 45.\cos 30$

$\Rightarrow 2.\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$

Örnek: $\cos 3x+\cos x = ?$

Çözüm: $\sin(90-3x)+\sin(90-x)$

$\Rightarrow 2.\sin\left(\frac{90-3x+90-x}{2}\right).\cos\left(\frac{90-3x-(90-x)}{2}\right)$

$\Rightarrow 2.\sin(90-2x).\cos(-x)$

$\Rightarrow 2.\cos 2x.\cos x$

$\sin a-\sin b = 2.\sin\left(\frac{a-b}{2}\right).\cos\left(\frac{a+b}{2}\right)$

Örnek: $\sin 105-\sin 15 = ?$

Çözüm: $2.\sin\left(\frac{105-15}{2}\right).\cos\left(\frac{105+15}{2}\right)$

$\Rightarrow 2.\sin 45.\cos 60 = 2.\frac{\sqrt{2}}{2}.\frac{1}{2} = \frac{\sqrt{2}}{2}$

2010-LYS: $\dfrac{1+\cos 40}{\cos 55 . \cos 35} = ?$

Çözüm: $1 = \cos 0$ yazılır.

$$\frac{\cos 0 + \cos 40}{\cos 55 . \cos 35} = \frac{\sin 90 + \sin 50}{\cos 55 . \cos 35}$$

$$= \frac{2 . \sin\left(\frac{90+50}{2}\right) . \cos\left(\frac{90-50}{2}\right)}{\frac{\cos(55+35)+\cos(55-35)}{2}}$$

$$= \frac{2 . \sin 70 . \cos 20}{\frac{\cos 90 + \cos 20}{2}} = \frac{2 . \sin 70 . \cancel{\cos 20}}{1} . \frac{2}{\cancel{\cos 20}}$$

$$= 4 \sin 70 \Rightarrow \boxed{4 . \cos 20^\circ}$$

1985-ÖYS: $\dfrac{1}{\sin 15} + \dfrac{1}{\cos 15} = ?$

Çözüm: $\dfrac{1}{\sin 15} + \dfrac{1}{\cos 15} = \dfrac{\cos 15 + \sin 15}{\sin 15 . \cos 15}$

(cos15) (sin15)

$$= \frac{\sin 75 + \sin 15}{2 \sin 15 \cos 15} = \frac{2 . \sin\left(\frac{75+15}{2}\right) . \cos\left(\frac{75-15}{2}\right)}{\frac{\sin 30}{2}}$$

$$= \frac{2 . \sin 45 . \cos 30}{\frac{\sin 30}{2}} = \frac{2 . \sin 45 . \cos 30}{1} . \frac{1}{\sin 30}$$

$$= \cancel{2} . \frac{\sqrt{2}}{\cancel{2}} . \frac{\sqrt{3}}{\cancel{2}} . \frac{\cancel{2}}{\frac{1}{2}} = \boxed{2\sqrt{6}}$$

1991-ÖYS: $\dfrac{\sin 3x}{\sin x} + \dfrac{\cos 3x}{\cos x} = 1$ ise $\cos^2 x = ?$

Çözüm: $\dfrac{\sin 3x}{\sin x} + \dfrac{\cos 3x}{\cos x} = \dfrac{\sin 3x . \cos x + \cos 3x . \sin x}{\sin x . \cos x}$

(cosx) (sinx)

$$\Rightarrow \frac{\sin(3x+x)}{2 . \sin x . \cos x} = \frac{\sin 4x}{\sin 2x} = \frac{2 . \sin 4x}{\sin 2x} = \frac{2 . 2 . \sin 2x \cos 2x}{\sin 2x}$$

$$\Rightarrow 4 . \cos 2x = 1 \text{ ise } \cos 2x = \frac{1}{4}$$

$$\cos^2 x - \underbrace{\sin^2 x}_{1-\cos^2 x} = \frac{1}{4}, \quad 2\cos^2 x - 1 = \frac{1}{4}$$

$$\boxed{\cos^2 x = \frac{5}{8}}$$

2007-ÖSS: $\dfrac{\sin 10 . \cos 40 + \cos 10 . \sin 40}{\cos 50 . \cos 10 + \sin 50 . \sin 10} = ?$

Çözüm: $\dfrac{\sin(10+40)}{\cos(50-10)} = \dfrac{\cancel{\sin 50}}{\cancel{\cos 40}} = \boxed{1}$

1996-ÖYS: $\dfrac{\sin 2A + \sin 4A}{\cos 2A + \cos 4A} = ?$

Çözüm:

$$\frac{\sin 2A + \sin 4A}{\sin(90-2A) + \sin(90-4A)}$$

$$= \frac{\cancel{2} . \sin\left(\frac{2A+4A}{2}\right) . \cos\left(\frac{2A-4A}{2}\right)}{\cancel{2} . \sin\left(\frac{90-2A+90-4A}{2}\right) . \cos\left(\frac{90-2A-(90-4A)}{2}\right)}$$

$$= \frac{\sin 3A . \cos(-A)}{\sin(90-3A) . \cos A} = \frac{\sin 3A}{\cos 3A} = \boxed{\tan 3A}$$

1996-ÖYS: $0 < \alpha < 90^\circ$

$$\frac{\sqrt{3} . \sin 5 . \cos 7 + \sqrt{3} . \cos 5 . \sin 7}{4 . \cos 84 . \cos 6} = \sin\alpha, \quad \alpha = ?$$

Çözüm:

$$\frac{\sqrt{3}\left(\sin 5 . \cos 7 + \cos 5 . \sin 7\right)}{2 . 2 . \underbrace{\sin 6 . \cos 6}_{\sin 12}} = \sin\alpha$$

$$= \frac{\sqrt{3} . \cancel{\sin 12}}{2 . \cancel{\sin 12}} = \sin\alpha \quad , \boxed{\alpha = 60^\circ}$$

1972-ÜSS: $\dfrac{\sin x + \sin 3x}{\cos x - \cos 3x} = ?$

Çözüm:

$$\frac{\sin x + \sin 3x}{\sin(90-x) - \sin(90-3x)}$$

$$\Rightarrow \frac{2 . \sin\left(\frac{x+3x}{2}\right) . \cos\left(\frac{x-3x}{2}\right)}{2 . \sin\left(\frac{90-x-(90-3x)}{2}\right) . \cos\left(\frac{90-x+90-3x}{2}\right)}$$

$$\Rightarrow \frac{\sin 2x . \cos(-x)}{\sin x . \cos(90-2x)}$$

$$\Rightarrow \boxed{\cot x}$$

1980-ÜSS: $10a = \frac{\pi}{2}$ ise $\frac{\cos 4a - \cos 8a}{\cos 4a . \cos 8a} = ?$

Çözüm: $10a = \frac{\pi}{2} \Rightarrow \cos 4a = \sin 6a$ ($4a+6a=10a$)

$\cos 8a = \sin 2a$ ($8a+2a=10a$)

Not: Sinüs ve cosinüsün 90° ye tamamlayan açıları birbirine eşittir!.

$$\frac{\sin 6a - \sin 2a}{\cos 4a . \cos 8a} = \frac{2.\sin\left(\frac{6a-2a}{2}\right).\cos\left(\frac{6a+2a}{2}\right)}{\cos 4a . \cos 8a}$$

$$\Rightarrow \frac{2.\sin 2a.\cos 4a}{\cos 4a.\cos 8a} = 2$$

Not:

- $\frac{\cos a + \cos 2a + \cos 3a}{\sin a + \sin 2a + \sin 3a} = \cot 2a$
- $\frac{\sin 3a + \sin 7a + \sin 11a}{\cos 3a + \cos 7a + \cos 11a} = \tan 7a$

1988-ÖYS: $\frac{\cos x + \cos 6x + \cos 11x}{\sin x + \sin 6x + \sin 11x} = ?$

Çözüm: Başta ve sondaki açıların toplamının yarısı ortadakini veriyorsa, pratik olarak sonuç: $\frac{\cos 6x}{\sin 6x} = \cot 6x$

Örnek: $\frac{1+\cos 35+\cos 70}{\sin 35+\sin 70} = ?$

Çözüm: $\frac{\cos 0 + \cos 35 + \cos 70}{\sin 0 + \sin 35 + \sin 70} = \cot 35°$

Örnek: $\pi = 8x$ ise, $\frac{\cos 13x + \cos 3x}{\sin 7x - \sin 3x} = ?$

Çözüm: $\pi = 8x$ ise $2\pi = 16x$ olur.

* $\cos 13x = \cos 3x$ (360° ye tamamladığından)

$$\frac{\cos 3x + \cos 3x}{2.\sin\left(\frac{7x-3x}{2}\right)\cos\left(\frac{7x+3x}{2}\right)} = \frac{2\cos 3x}{2.\sin 2x.\cos 5x}$$

* $\cos 3x = -\cos 5x$ (180° ye tamamladığından)
* $\sin 2x = \sin 4S$

$$\Rightarrow \frac{2.\cos 3x}{2.\sin 4S.\cos 5x} \cdot (-1) = \frac{-1}{\frac{1}{\sqrt{2}}} = -\sqrt{2}$$

Örnek: $\sin 20 . \sin 40 . \sin 60 . \sin 80 = ?$

Çözüm: $\cos 70 . \cos 50 . \cos 30 . \cos 10$

$\Rightarrow \cos 70 . \cos 10 . \cos 50 . \cos 30$

$\Rightarrow \frac{\cos 80 + \cos 60}{2} . \frac{\cos 80 + \cos 20}{2}$

$\Rightarrow \frac{1}{4}\left[(\cos 80 + \cos 60).(\cos 80 + \cos 20)\right]$

$= \frac{1}{4}[(\cos 80.\cos 80 + \cos 80.\cos 20 + \cos 60.\cos 80 + \cos 60.\cos 20)]$

$= \frac{1}{4}\left[\frac{\cos 160 + \cos 0}{2} + \frac{\cos 100 + \cos 60}{2} + \frac{\cos 140 + \cos 20}{2} + \frac{\cos 80 + \cos 40}{2}\right]$

$= \frac{1}{8}\left[-\cos 20 + 1 - \cos 80 + \frac{1}{2} - \cos 40 + \cos 20 + \cos 80 + \cos 40\right]$

$= \frac{1}{8}.\left(1 + \frac{1}{2}\right) = \frac{1}{8}.\frac{3}{2} = \frac{3}{16}$

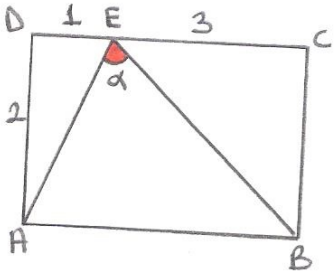
Örnek: $\sin 20 . \sin 40 . \sin 80 = ?$

Çözüm: $\sin 20 . \sin 40 . \sin 60 . \sin 80$ / $\sin 60$

$= \frac{\frac{3}{16}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{8}$

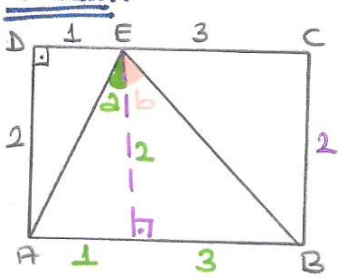
$\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a . \tan b}$

Örnek:



ABCD dikdörtgen ise $\tan\alpha$ kaçtır?

Çözüm:



$\alpha = a+b$ ise

$\tan\alpha = \tan(a+b)$

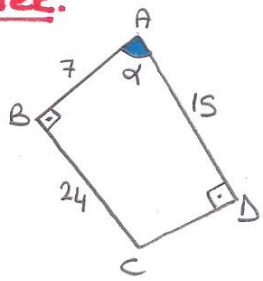
$* \tan a = \frac{1}{2}$

$* \tan b = \frac{3}{2}$

$$\tan\alpha = \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a . \tan b} = \frac{\frac{1}{2}+\frac{3}{2}}{1-\frac{1}{2}\cdot\frac{3}{2}}$$

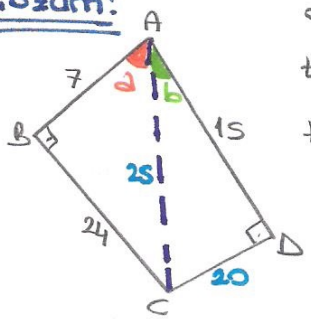
$$\Rightarrow \frac{2}{1-\frac{3}{4}} = \frac{2}{\frac{1}{4}} = \boxed{8}$$

Örnek:



$\tan\alpha$ kaçtır?

Çözüm:



$\alpha = a+b$ ise

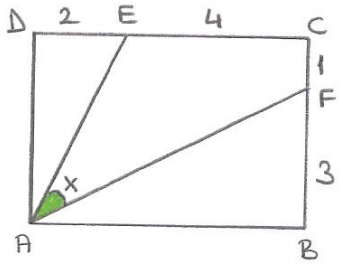
$\tan\alpha = \tan(a+b)$

$\tan a = \frac{24}{7}$, $\tan b = \frac{20}{15} = \frac{4}{3}$

$\tan\alpha = \tan(a+b)$

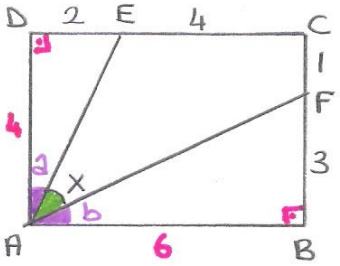
$$\Rightarrow \frac{\frac{24}{7}+\frac{4}{3}}{1-\frac{24}{7}\cdot\frac{4}{3}} = \boxed{\frac{-4}{3}}$$

Örnek:



ABCD dikdörtgen ise $\tan x = ?$

Çözüm:



$x + a + b = 90°$

$x = 90° - (a+b)$

$\tan x = \tan(90 - (a+b))$

$\tan x = \cot(a+b)$

$$\tan x = \frac{1}{\tan(a+b)}$$

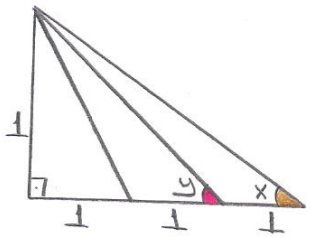
$\tan a = \frac{2}{4} = \frac{1}{2}$

$\tan b = \frac{3}{6} = \frac{1}{2}$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a . \tan b} = \frac{\frac{1}{2}+\frac{1}{2}}{1-\frac{1}{2}\cdot\frac{1}{2}} = \frac{4}{3}$$

$$\tan x = \frac{1}{\tan(a+b)} = \boxed{\frac{3}{4}}$$

Örnek:



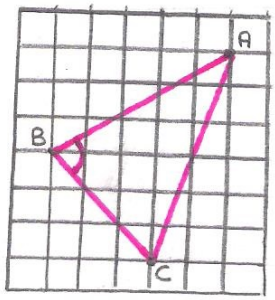
$x+y$ kaçtır?

Çözüm: $\tan x = \frac{1}{3}$, $\tan y = \frac{1}{2}$

$$\tan(x+y) = \frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\cdot\frac{1}{2}} = \frac{\frac{5}{6}}{1-\frac{1}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

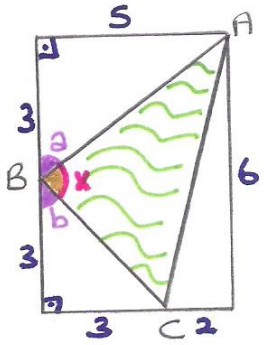
$\tan(x+y) = 1$ ise $\boxed{x+y = 45°}$

2011-LYS:



Birim kareler üzerine çizilmiş yandaki ABC üçgeninin B açısının tanjantı kaçtır?

Çözüm: Şekil, dikdörtgene tamamlanırsa,



$x+a+b=180°$

$x=180°-(a+b)$

$\tan x=\tan(180-(a+b))$

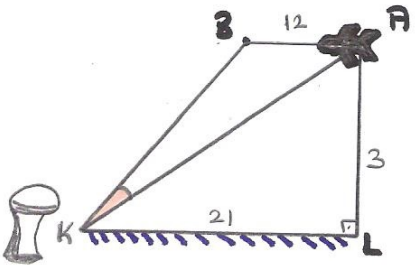
$\tan x=-\tan(a+b)$

- $\tan a=\frac{5}{3}$
- $\tan b=\frac{3}{3}=1$

$$\tan(a+b)=\frac{\frac{5}{3}+1}{1-\frac{5}{3}\cdot 1}=\frac{\frac{8}{3}}{1-\frac{5}{3}}=\frac{\frac{8}{3}}{\frac{-2}{3}}=-4$$

$\tan x=-\tan(a+b)=--4=4$

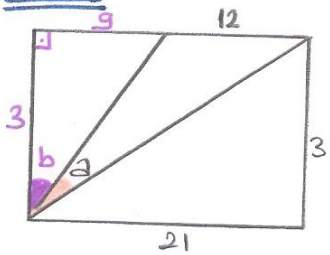
2006-ÖSS:



AB // KL

$\tan(\hat{AKB})=?$

Çözüm:



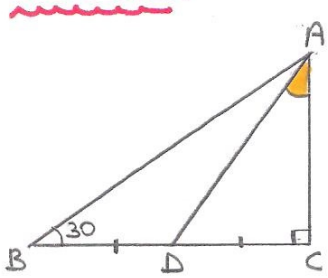
- $\tan(a+b)=\frac{21}{3}=7$
- $\tan b=\frac{9}{3}=3$

$$\tan(a+b)=\frac{\tan a+\tan b}{1-\tan a.\tan b}$$

$$\frac{7}{1}=\frac{\tan a+3}{1-3.\tan a}$$

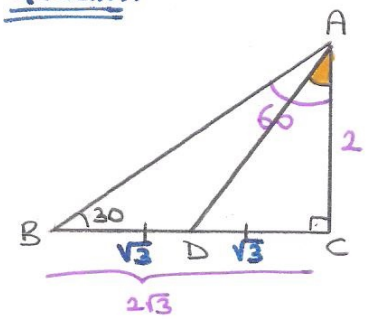
$\tan a=\frac{2}{11}$

1983-ÖYS:



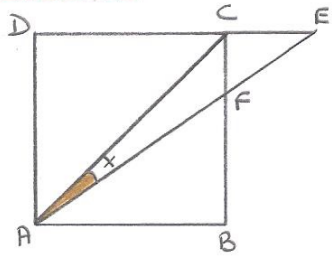
$\tan(\hat{DAC})=?^\circ$

Çözüm:



$\tan(\hat{DAC})=\frac{\sqrt{3}}{2}$

1978-ÜSS:

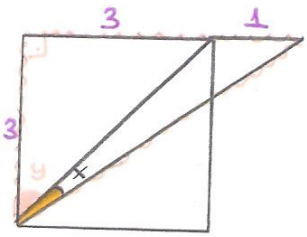


ABCD kare

$|CE|=\frac{|DC|}{3}$ ise

$\tan x=?^\circ$

Çözüm:



$\tan(x+y)=\frac{4}{3}$

$\tan y=\frac{3}{3}=1$

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x.\tan y}$$

$\frac{4}{3}=\frac{\tan x+1}{1-1.\tan x}$, $4-4\tan x=3\tan x+3$

$1=7\tan x$, $\tan x=\frac{1}{7}$

#cyhnyvz#

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Başarıya açılan pencereniz...

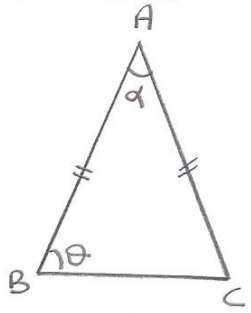
$\tan(a-b) = \dfrac{\tan a - \tan b}{1+\tan a.\tan b}$

1970-ÜSS: $\dfrac{\tan 60 - \tan 30}{1+\tan 60.\tan 30} = ?$

Çözüm: $\tan(60-30) = \tan 30 = \dfrac{\sqrt{3}}{3}$

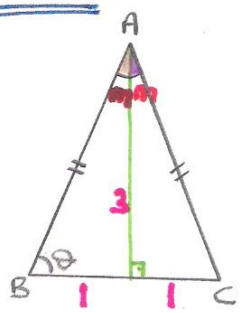
$\tan 2a = \dfrac{2\tan a}{1-\tan^2 a}$

1997-ÖSS:



$\tan\theta = 3$ ise $\tan\alpha = ?$

Çözüm:



$\alpha = 2m$ olsun.

$\tan\alpha = \tan 2m$

$\tan m = \dfrac{1}{3}$ ise

$\tan\alpha = \tan 2m = \dfrac{2\tan m}{1-\tan^2 m}$

$\tan\alpha = \dfrac{2.\frac{1}{3}}{1-\frac{1}{9}} = \dfrac{\frac{2}{3}}{\frac{8}{9}} = \dfrac{3}{4}$

Uyarı: $a.\cos x \mp b.\sin x$ ifadesinin en büyük ve en küçük değerleri: $\mp\sqrt{a^2+b^2}$

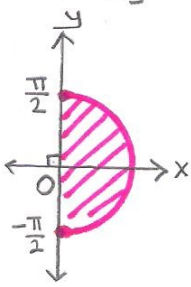
Örnek: $3\sin x - 4\cos x$ ifadesinin alacağı en büyük değer nedir?

Çözüm: $\sqrt{3^2+(-4)^2} = \sqrt{25} = 5$

Ters Trigonometrik fonksiyonlar:

arcsin fonksiyonu:

$\sin: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1,1]$



arcsin fonksiyonu $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ aralığında açı değeri alır, değer aralığı $[-1,1]$ arasındadır.

$y = \sin x \longrightarrow x = \arcsin y$

Yorum: $\arcsin x$ demek, hangi açının sinüs değeri x'e eşit demektir. Unutulmamalıdır ki sinüs ve tanjant için x pozitif ise açı 1. bölgede, negatif ise 4. bölgede alınacaktır.

Örnek: $\arcsin\frac{\sqrt{3}}{2} + \arcsin\frac{1}{2} = ?$

Çözüm: $\arcsin\frac{\sqrt{3}}{2} = 60^\circ$, $\arcsin\frac{1}{2} = 30^\circ$

$30^\circ + 60^\circ = 90^\circ$

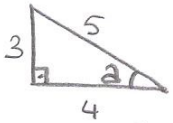
Örnek: $\sin(\arcsin(-1)) = ?$

Çözüm: $\arcsin(-1) = 270^\circ$, $\sin 270 = -1$

Örnek: $\cos\left(\arcsin\frac{3}{5}\right) = ?$

Çözüm: $\arcsin\frac{3}{5} = a$ ise $\sin a = \frac{3}{5}$

$\cos\underbrace{\left(\arcsin\frac{3}{5}\right)}_{a} \longrightarrow \cos a = ?$

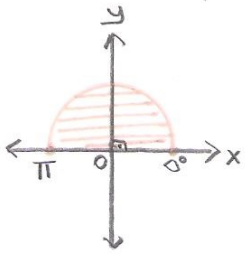


$\cos a = \frac{4}{5}$

$\sin a = \frac{3}{5}$

arccos fonksiyonu:

$\cos : [0,\pi] \longrightarrow [-1,1]$



arccos fonksiyonu $[0,\pi]$ aralığında açı değeri alır. Değer aralığı da $[-1,1]$ dir.

$x = \cos y \longrightarrow y = \arccos x$

Yorum: arccosx ve arccotx için bulacağımız açı değerleri için x pozitif ise açı 1. bölgede, negatif ise 2. bölgede alınır.

Örnek: $\arccos\left(-\frac{1}{2}\right) + \arccos\left(-\frac{\sqrt{2}}{2}\right) = ?$

Çözüm: $\arccos\left(-\frac{1}{2}\right) = 120°$, $\arccos\left(\frac{-\sqrt{2}}{2}\right) = 135°$ } $120+135 = 255°$

Örnek: $\sin\left(\arccos\frac{4}{5} + \arccos\frac{5}{13}\right) = ?$

Çözüm: $\sin\left(\underbrace{\arccos\frac{4}{5}}_{a} + \underbrace{\arccos\frac{5}{13}}_{b}\right) = \sin(a+b) = ?$

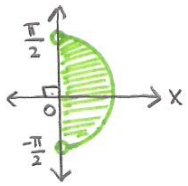
$\cos a = \frac{4}{5}$ $\quad \cos b = \frac{5}{13}$

$\sin(a+b) = \sin a.\cos b + \sin b.\cos a$

$= \frac{3}{5}.\frac{5}{13} + \frac{12}{13}.\frac{4}{5} = \frac{15+48}{65} = \frac{63}{65}$

arctan fonksiyonu:

$\tan : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \longrightarrow \mathbb{R}$



$x = \arctan y$

$\tan x = y$

Örnek: $4\arctan 1 - 3\arctan\sqrt{3} = ?$

Çözüm: $\arctan 1 = 45°$, $\arctan\sqrt{3} = 60°$

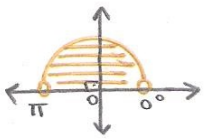
$4\arctan 1 - 3\arctan\sqrt{3} = 4.45 - 3.60 = 0$

Örnek: $\arctan(-1) + \arctan\left(-\frac{1}{\sqrt{3}}\right) = ?$

Çözüm: $\arctan(-1) = 315°$, $\arctan\left(-\frac{1}{\sqrt{3}}\right) = 330°$ } $315+330 = 645°$

arccot fonksiyonu:

$\cot : (0,\pi) \longrightarrow \mathbb{R}$



$x = \cot y \Rightarrow y = \operatorname{arccot} x$

Örnek: $\operatorname{arccot}\sqrt{3} + \operatorname{arccot} 1 = ?$

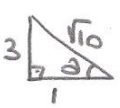
Çözüm: $\operatorname{arccot}\sqrt{3} = 30°$, $\operatorname{arccot} 1 = 45°$ } $30+45 = 75$

Örnek: $\sin(2\arctan 3) = ?$

Çözüm: $\sin(\underbrace{2\arctan 3}_{a}) = \sin(2a) = ?$

$\arctan 3 = a$

$\tan a = 3$



$\sin 2a = 2.\sin a.\cos a$

$\sin 2a = 2.\frac{3}{\sqrt{10}}.\frac{1}{\sqrt{10}} = \frac{6}{10} = \frac{3}{5}$

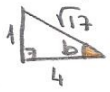
Örnek: $\text{arccot}\frac{1}{4} - \arctan\frac{1}{4} = \arccos x$ ise x kaçtır?

Çözüm: $\underbrace{\text{arccot}\frac{1}{4}}_{a} - \underbrace{\arctan\frac{1}{4}}_{b} = \arccos x$

$a-b = \arccos x \rightarrow x = \cos(a-b)$

$\text{arccot}\frac{1}{4} = a$ $\quad \arctan\frac{1}{4} = b$

$\cot a = \frac{1}{4}$ $\quad \tan b = \frac{1}{4}$



$x = \cos(a-b) = \cos a.\cos b + \sin a.\sin b$

$x = \frac{1}{\sqrt{17}}\cdot\frac{4}{\sqrt{17}} + \frac{4}{\sqrt{17}}\cdot\frac{1}{\sqrt{17}} = \frac{4}{17} + \frac{4}{17} = \boxed{\frac{8}{17}}$

2011-LYS: $f(x) = \arcsin\left(\frac{x}{3}+2\right)$ ise $f^{-1}(x)$ nedir?

Çözüm: $f^{-1}\underbrace{\left(\arcsin\left(\frac{x}{3}+2\right)\right)}_{y} = x$

$\arcsin\left(\frac{x}{3}+2\right) = y$ ise $\sin y = \frac{x}{3}+2$

$\frac{x}{3} = \sin y - 2$, $x = 3\sin y - 6$

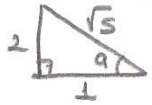
$f^{-1}(y) = 3\sin y - 6 \rightarrow \boxed{f^{-1}(x) = 3\sin x - 6}$

1995-ÖYS: $\cos(2\text{arccot}\frac{1}{2}) = ?$

Çözüm: $\cos(\underbrace{2\text{arccot}\frac{1}{2}}_{a}) = \cos 2a = ?$

$\text{arccot}\frac{1}{2} = a$

$\cot a = \frac{1}{2}$



$\cos 2a = \cos^2 a - \sin^2 a$

$= \left(\frac{1}{\sqrt{5}}\right)^2 - \left(\frac{2}{\sqrt{5}}\right)^2$

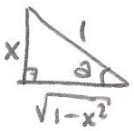
$= \frac{1}{5} - \frac{4}{5} = \boxed{\frac{-3}{5}}$

1976-ÜSS: $\cos(\arcsin x) = ?$

Çözüm: $\cos(\underbrace{\arcsin x}_{a}) = \cos a = ?$

$\arcsin x = a$

$\sin a = x$



$\cos a = \sqrt{1-x^2}$

#cyhnyvz#

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Başarıya açılan pencereniz...

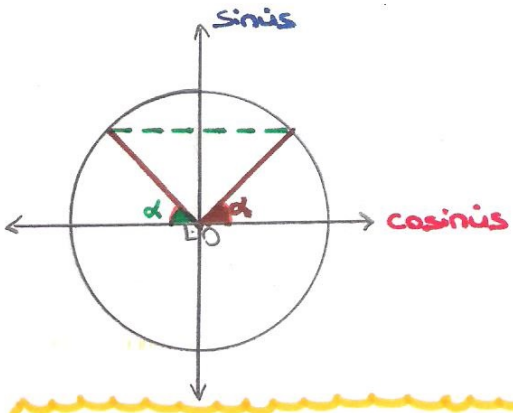
Trigonometri II

Trigonometrik Denklemler:

I. Tür Denklemler:

1) Sinüs Fonksiyonu:

$0 \le \alpha < 2\pi$ olmak üzere $\sin x = \sin\alpha$ denkleminin çözümü:



$x_1 = \alpha + 2k\pi \quad , \quad x_2 = (\pi - \alpha) + 2k\pi$

$$Ç = \{ x \mid x_1 = \alpha + 2k\pi \ \vee \ x_2 = (\pi - \alpha) + 2k\pi \ , \ k \in \mathbb{Z} \}$$

Örnek: $\sin x = 0$ denkleminin Ç.K = ?

Çözüm: $Ç = \{ x \mid x = k\pi, \ k \in \mathbb{Z} \}$

Örnek: $0 \le x < 2\pi$, $\sin^2 x - 3\sin x + 2 = 0$ denkleminin Ç.K = ?

Çözüm: $\sin x = t$ için, $t^2 - 3t + 2 = 0$

$t \quad -2$

$t \quad -1$

$t = 1 \Rightarrow \sin x = 1$

$x_1 = \frac{\pi}{2} \ \vee \ x_2 = \pi - \frac{\pi}{2} = \frac{\pi}{2}$

$t = 2 \Rightarrow \sin x = 2$ için kök yoktur.

$$Ç = \left\{ \frac{\pi}{2} \right\}$$

Örnek: $\sin 2x = \sin\left(\frac{\pi}{2} + x\right)$ için Ç.K = ?

Çözüm:

$2x = \frac{\pi}{2} + x + 2k\pi \ \vee \ 2x = \pi - \left(\frac{\pi}{2} + x\right) + 2k\pi$

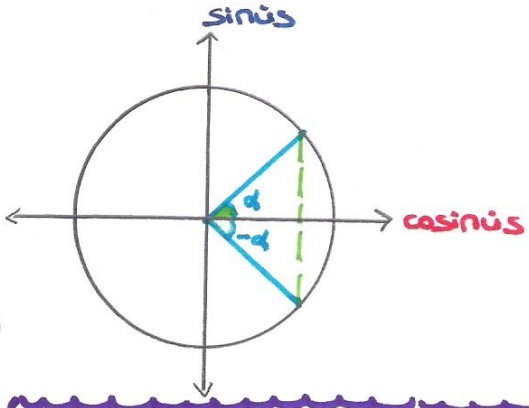
$x = \frac{\pi}{2} + 2k\pi \ \vee \ 3x = \frac{\pi}{2} + 2k\pi$

$x = \frac{\pi}{6} + \frac{2k\pi}{3}$

$$Ç = \left\{ x \mid x = \frac{\pi}{2} + 2k\pi \ \vee \ x = \frac{\pi}{6} + \frac{2k\pi}{3} \ , \ k \in \mathbb{Z} \right\}$$

2) Cosinüs Fonksiyonu:

$0 \le \alpha < 2\pi$ olmak üzere $\cos x = \cos\alpha$ denkleminin çözümü:



$x_1 = \alpha + 2k\pi \quad , \quad x_2 = -\alpha + 2k\pi$

$$Ç = \{ x \mid x_1 = \alpha + 2k\pi \ \vee \ x_2 = -\alpha + 2k\pi \ , \ k \in \mathbb{Z} \}$$

Örnek: $\cos x = -1$ denkleminin Ç.K = ?

Çözüm: π nin tek katlarında cosinüs -1 olduğundan,

$x = \pi + 2k\pi \quad , \quad x = \pi(2k+1)$

$$Ç = \{ x = \pi(2k+1) \ , \ k \in \mathbb{Z} \}$$

Örnek: $2\cos^2 2x - 1 = 0$ denkleminin Ç.K = ?

Çözüm:

$2\cos^2 2x = 1$ ise $\cos^2 2x = \frac{1}{2}$, $\cos 2x = \mp \frac{1}{\sqrt{2}}$

* $\cos 2x = \frac{1}{\sqrt{2}}$ ise

$2x = \frac{\pi}{4} + 2k\pi$ ∨ $2x = -\frac{\pi}{4} + 2k\pi$

$x = \frac{\pi}{8} + k\pi$ ∨ $x = -\frac{\pi}{8} + k\pi$

* $\cos 2x = -\frac{1}{\sqrt{2}}$ ise

$2x = \frac{3\pi}{4} + 2k\pi$ ∨ $2x = -\frac{3\pi}{4} + 2k\pi$

$x = \frac{3\pi}{8} + k\pi$ ∨ $x = -\frac{3\pi}{8} + k\pi$

$Ç = \left\{ x \mid x = \mp\frac{\pi}{8} + k\pi \vee x = \mp\frac{3\pi}{8} + k\pi, k \in \mathbb{Z} \right\}$

Örnek: $\cos 2x = -\frac{1}{2}$ nin $[0, 2\pi)$ aralığındaki çözüm kümesi nedir?

Çözüm: $2x = \frac{2\pi}{3} + 2k\pi$ ∨ $2x = -\frac{2\pi}{3} + 2k\pi$

$x = \frac{\pi}{3} + k\pi$ ∨ $x = -\frac{\pi}{3} + k\pi$

$k = 0$ ise $x = \frac{\pi}{3}$

$k = 1$ ise $x = \frac{4\pi}{3}$ ve $x = \frac{2\pi}{3}$

$k = 2$ ise $x = \frac{5\pi}{3}$

$Ç: \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$

3) tan ve cot fonksiyonları:

$0 \le \alpha < \pi$ olmak üzere, $\cot x = \cot\alpha$, $\tan x = \tan\alpha$

denk. çözümü:

- $\tan x = \tan\alpha$ ise $x = \alpha + k\pi$, $k \in \mathbb{Z}$
- $\cot x = \cot\alpha$ ise $x = \alpha + k\pi$, $k \in \mathbb{Z}$

Örnek: $\tan x = -1$ denkleminin Ç.K = ?

Çözüm: $\tan x = -1$ ise x in en küçük açı değeri 135° dir. O halde,

$x = 135 + 180k$, $k \in \mathbb{Z}$, veya $135° = \frac{3\pi}{4}$ old.

$Ç = \left\{ x \mid x = \frac{3\pi}{4} + k\pi, k \in \mathbb{Z} \right\}$

Örnek: $3\cot x = \sqrt{3}$ denkleminin $[0, 2\pi)$ aralığındaki Ç.K = ?

Çözüm: $\cot x = \frac{\sqrt{3}}{3}$ ise,

$x = 60 + 180k$, $k \in \mathbb{Z}$ veya

$x = \frac{\pi}{3} + k\pi$, $k \in \mathbb{Z}$ şeklinde yazılır.

$k = 0$ için $x = 60°$

$k = 1$ için $x = 240$

$Ç = \{60, 240\}$

$Ç = \left\{ \frac{\pi}{3}, \frac{4\pi}{3} \right\}$

Örnek: $\tan^2 3x - 3 = 0$ denkleminin $[0, 2\pi)$ aralığındaki en küçük pozitif kökü kaç radyandır?

Çözüm: $\tan^2 3x = 3$, $\tan 3x = \mp\sqrt{3}$

- $\tan 3x = \sqrt{3}$ ise $3x = \frac{\pi}{3} + k\pi$, $x = \frac{\pi}{9} + \frac{k\pi}{3}$
- $\tan 3x = -\sqrt{3}$ ise $3x = \frac{2\pi}{3} + k\pi$, $x = \frac{2\pi}{9} + \frac{2k\pi}{3}$

$k = 0$ için $x = \frac{\pi}{9}$ veya $x = \frac{2\pi}{9}$

Küçük açı: $\frac{\pi}{9}$

II. Tür Denklemler:

Örnek: $\sin 3x = \cos\left(\frac{\pi}{3}+x\right)$ denkleminin Ç.K=?

Çözüm: $\cos\left(\frac{\pi}{3}+x\right)$ ifadesi sinüse çevrilir.

$a+b=90^\circ$ ise $\cos a = \sin b$ olduğundan $\cos(90-a)=\sin a$ olur. Buna göre,

$$\cos\left(\frac{\pi}{3}+x\right)=\sin\left[\frac{\pi}{2}-\left(\frac{\pi}{3}+x\right)\right]=\sin\left(\frac{\pi}{6}-x\right)$$

O halde,

$$\sin 3x=\sin\left(\frac{\pi}{6}-x\right)$$

$3x=\frac{\pi}{6}-x+2k\pi$ ∨ $3x=\pi-\left(\frac{\pi}{6}-x\right)+2k\pi$

$4x=\frac{\pi}{6}+2k\pi$ ∨ $2x=\frac{5\pi}{6}+2k\pi$

$x=\frac{\pi}{24}+\frac{k\pi}{2}$ ∨ $x=\frac{5\pi}{12}+k\pi$

$$Ç: \left\{x \mid x=\frac{\pi}{24}+\frac{k\pi}{2} \vee x=\frac{5\pi}{12}+k\pi,\ k\in\mathbb{Z}\right\}$$

Örnek: $\tan x.\tan 3x = 1$ denkleminin $[0,2\pi)$ aralığındaki en küçük kökü?

Çözüm:

$\tan x.\tan 3x=1$ ise $\tan x=\frac{1}{\tan 3x}=\cot 3x$

$\tan x=\cot 3x \rightarrow \tan(90-3x)$

$\tan x=\tan(90-3x)$

$x=90-3x+180k$, $k\in\mathbb{Z}$

$4x=90+180k$, $k\in\mathbb{Z}$

$x=22{,}5+45k$, $k\in\mathbb{Z}$

k=0 için pozitif en küçük kök: 22,5

Örnek: $2\sin^2 x-\sin x=0$ denkleminin $[0,\pi]$ aralığındaki Ç.K nedir?

Cevap: $\{0, 30, 150, 180\}$

Örnek: $\tan\left(x+\frac{\pi}{12}\right)=1$ denkleminin en küçük pozitif kökü nedir?

Cevap: $\left\{\frac{\pi}{6}\right\}$

III. Tür Denklemler:

Örnek: $\sqrt{3}\sin x+\cos x=2$ için Ç.K=?

Çözüm: $\sqrt{3}=\tan 60$ yazılırsa,

$\tan 60.\sin x+\cos x=2$

$\frac{\sin 60}{\cos 60}.\sin x+\cos x=2$ /.cos60

$\sin 60.\sin x+\cos 60.\cos x=2.\cos 60.\frac{1}{2}=1$

$\cos(x-60)=1=\cos 0$

$x-60=0+360k$, $x=60+360k$

$$Ç: \left\{x \mid x=\frac{\pi}{3}+2k\pi,\ k\in\mathbb{Z}\right\}$$

Örnek: $3\sin x-\sqrt{3}\cos x=\sqrt{3}$ denkleminin $[0,2\pi]$ aralığındaki Ç.K nedir?

Çözüm: Her tarafı $\frac{1}{3}$ ile çarpalım.

$\sin x-\frac{\sqrt{3}}{3}.\cos x=\frac{\sqrt{3}}{3}$, $\frac{\sqrt{3}}{3}=\tan 30$ yazalım.

$\sin x-\tan 30.\cos x=\frac{\sqrt{3}}{3}$

$\sin x-\frac{\sin 30}{\cos 30}.\cos x=\frac{\sqrt{3}}{3}$

$\sin x.\cos 30-\sin 30.\cos x=\frac{\sqrt{3}}{3}.\cos 30.\frac{\sqrt{3}}{2}$

$\sin(x-30)=\frac{1}{2}$

$x-30=30+2k\pi$, $x-30=150+2k\pi$

$x=60+2k\pi$, $x=180+2k\pi$

k=0 için

$$Ç: \{60, 180\}$$

4. Tür Homojen Denklemler:

Örnek: $\sin x+\sqrt{3}.\cos x=0$ denk. Ç.K=?

Çözüm:

$\sin x=-\sqrt{3}.\cos x$, $\tan x=-\sqrt{3}$

$x=120+180k$, $k\in\mathbb{Z}$

$Ç.K=\left\{ x \mid x=\frac{2\pi}{3}+k\pi,\ k\in\mathbb{Z} \right\}$

Örnek: $2\sin x-3\cos x=0$ denk. Ç.K=?

Çözüm: $2\sin x=3\cos x$, $\tan x=\frac{3}{2}$

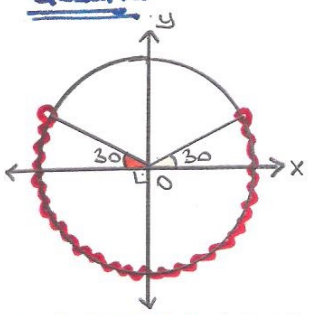
$x=\arctan\frac{3}{2}+k\pi$

$Ç=\left\{ x \mid x=\arctan\frac{3}{2}+k\pi,\ k\in\mathbb{Z} \right\}$

Trigonometrik Eşitsizlikler

Örnek: $\sin x<\frac{1}{2}$,

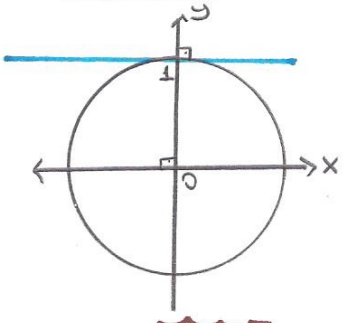
Çözüm:



$[0,30)\cup(150,360]$

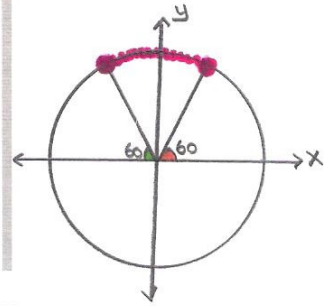
Örnek: $\sin x\geq 1$

Çözüm:



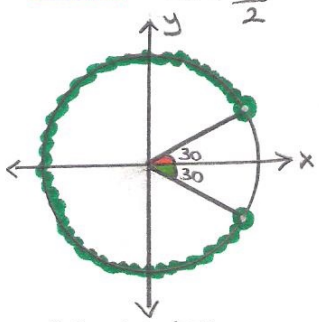
$\left\{\frac{\pi}{2}\right\}$

Örnek: $\sin x\geq\frac{\sqrt{3}}{2}$



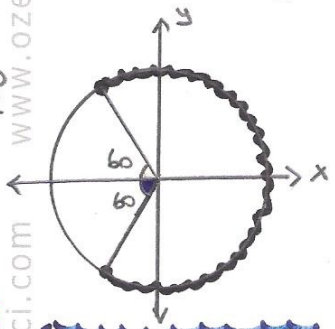
$[60,120]$

Örnek: $\cos x<\frac{\sqrt{3}}{2}$



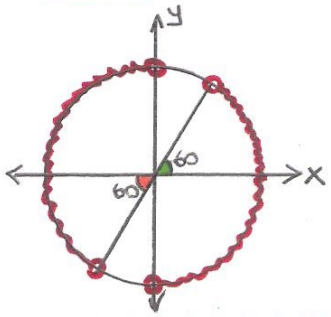
$(30,330)$

Örnek: $\cos x\geq-\frac{1}{2}$



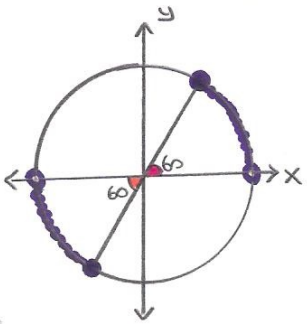
$[0,120]\cup[240,360]$

Örnek: $\tan x<\sqrt{3}$



$[0,60)\cup(90,240)\cup(270,360]$

Örnek: $\cot x\geq\frac{\sqrt{3}}{3}$



0° ve 180° de tanımsız old.

$(0,60]\cup(180,240]$

Periyodik Fonksiyon:

$a, b \in \mathbb{R}$ ve $n \in \mathbb{Z}^+$ olmak üzere,

n tek ise,

- $\sin^n(ax+b)$, $\cos^n(ax+b)$ } için periyot = $T = \frac{2\pi}{|a|}$

n çift ise,

- $\sin^n(ax+b)$, $\cos^n(ax+b)$ } için periyot = $T = \frac{\pi}{|a|}$

$m \in \mathbb{Z}^+$ ise,

- $\tan^m(ax+b)$, $\cot^m(ax+b)$ } için periyot = $T = \frac{\pi}{|a|}$

Örnek: $f(x) = \cos^2(3x+1)$ in periyodu?

Cevap: $\frac{\pi}{3}$

Örnek: $f(x) = \sin^3\frac{\pi x}{2}$ nin periyodu?

Cevap: $\frac{2\pi}{\frac{\pi}{2}} = 4$

Not: Toplam biçiminde verilen fonksiyonların periyodunu bulmak için ayrı ayrı periyotlar bulunup OKEK'i alınır.

Örnek: $f(x) = 3\sin^3\frac{x}{2} + \tan^2 2x$ periyodu?

Çözüm: $T_1 = \frac{2\pi}{\frac{1}{2}} = 4\pi$, $T_2 = \frac{\pi}{2}$

$T = (4\pi, \frac{\pi}{2})_{okek} = \boxed{4\pi}$

Not: Çarpım durumundaki fonksiyonların periyodunu bulmak için ifade toplam veya fark durumuna getirilir.

Örnek: $f(x) = 3.\sin(2x+1).\cos(3x-1)$ periyodu?

Çözüm: $3.\frac{1}{2}.[\underbrace{\sin 5x}_{T_1=\frac{2\pi}{5}} - \underbrace{\sin(x-2)}_{T_2=2\pi}]$

$T = (T_1, T_2)_{okek} = (\frac{2\pi}{5}, 2\pi)_{okek} = \boxed{2\pi}$

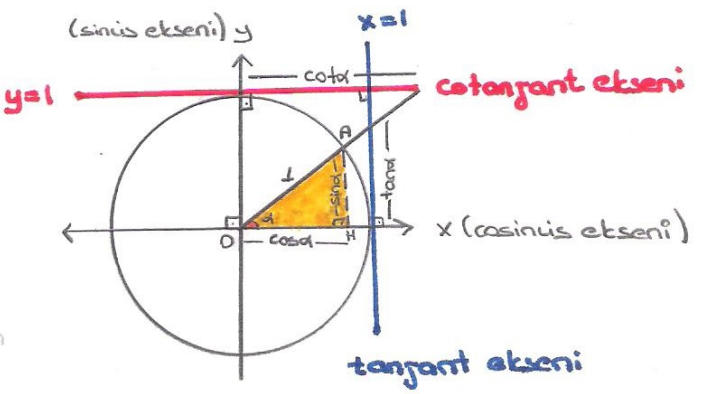
Örnek: $f(x) = \cos 2x.\cos(2x+\frac{\pi}{3})$ ifadesinin periyodu kaçtır?

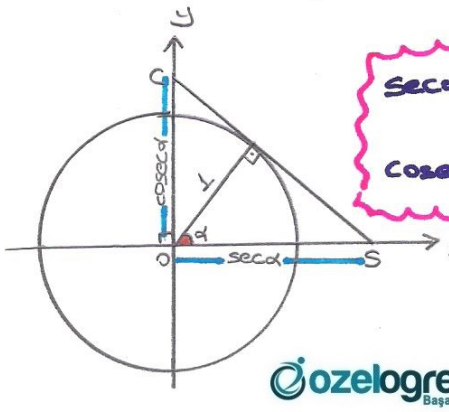
Çözüm: $f(x) = \frac{1}{2}[\cos(4x+\frac{\pi}{3}) + \cos(-\frac{\pi}{3})]$

$= \frac{1}{2}[\cos(4x+\frac{\pi}{3}) + \frac{1}{2}]$ olup

$\boxed{T = \frac{2\pi}{4} = \frac{\pi}{2}}$

Trigonometrik Fonksiyonların Grafikleri



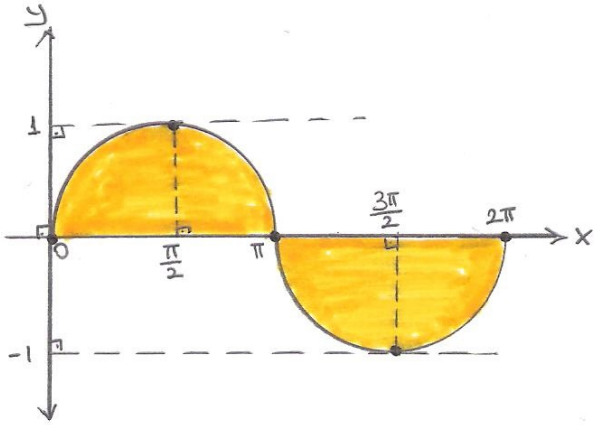


$\sec\alpha = \frac{1}{\cos\alpha}$

$\text{Cosec}\alpha = \frac{1}{\sin\alpha}$

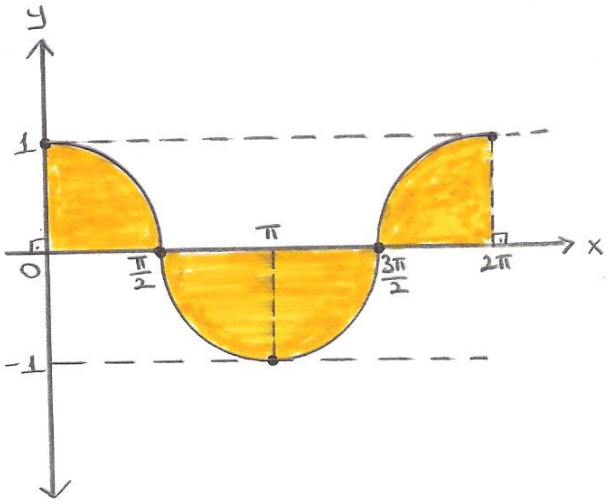
Sinüs Fonksiyonunun Grafiği:

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sinx	0	1	0	1	0

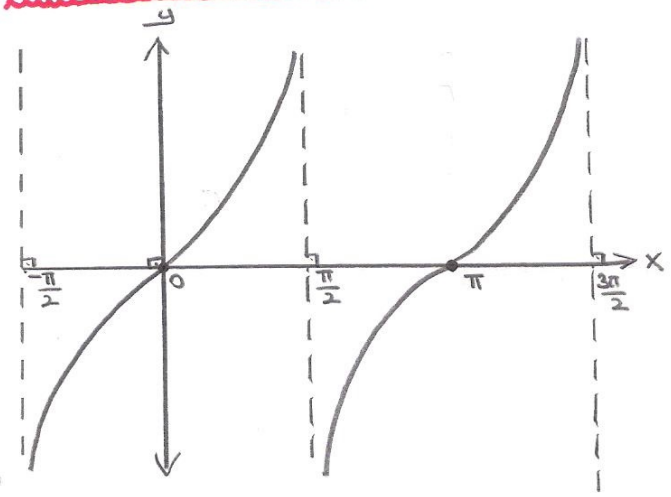


cosinüs Fonksiyonunun Grafiği:

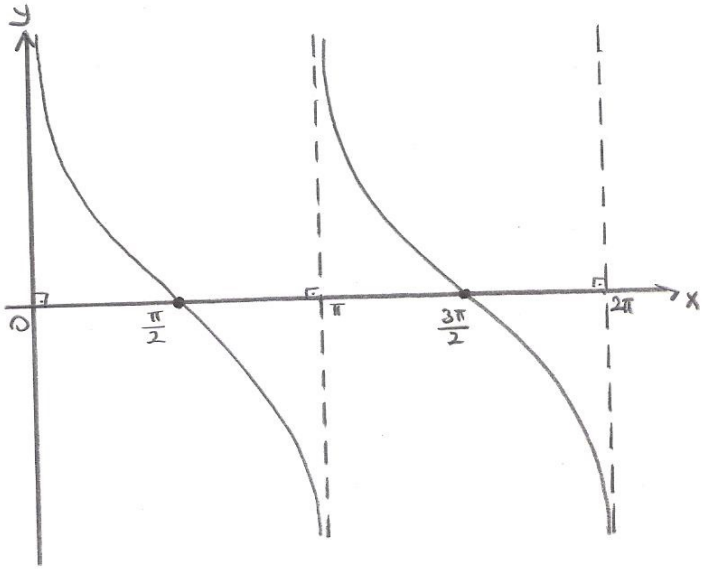
0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
1	0	-1	0	1



tanjant fonksiyonunun Grafiği:



cotanjant Fonksiyonunun Grafiği:



Sayfa: CC6

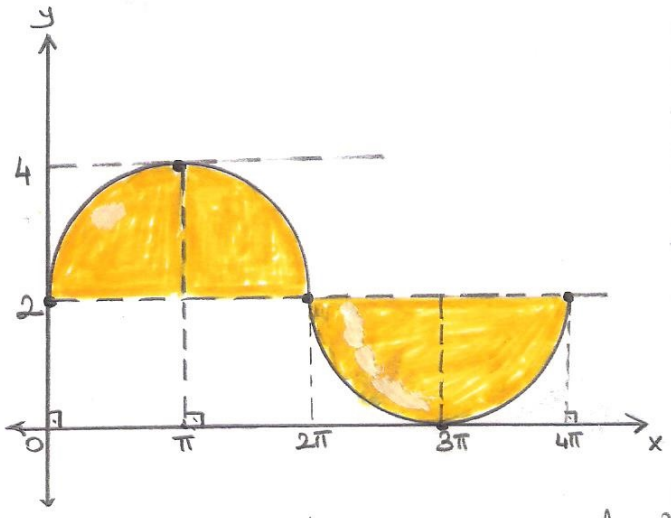
#cyhnyz#

Alıştırmalar:

1) $f(x)=2+2\sin\frac{x}{2}$ fonksiyonunun grafiği?

Çözüm: periyot $= T = \frac{2\pi}{\frac{1}{2}} = 4\pi$

x	0	π	2π	3π	4π
$\sin\frac{x}{2}$	0	1	0	-1	0
$2\sin\frac{x}{2}$	0	2	0	-2	0
$2+2\sin\frac{x}{2}$	2	4	2	0	2



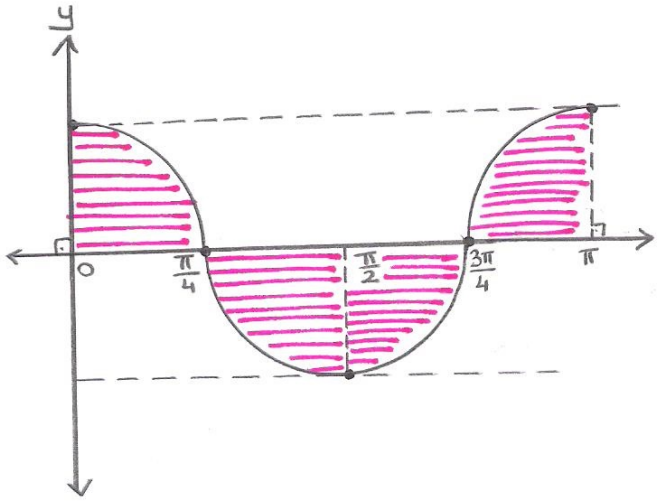
2) $f(x)=2\cos^2x-1$ fonksiyonunun grafiği?

Çözüm: $2\cos^2x-1 = \cos^2x+\cos^2x-(\sin^2x+\cos^2x)$

$\Rightarrow \cos^2x-\sin^2x=\cos2x$

periyot $= \pi = \frac{2\pi}{2} = \pi$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$\cos2x$	1	0	-1	0	1



3) $f(x)=\sin x.\cos x$ fonksiyonunun grafiği?

Çözüm: $\sin x.\cos x = \frac{\sin2x}{2}$, periyot $= \frac{2\pi}{2}=\pi$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$\sin2x$	0	1	0	-1	0
$\frac{\sin2x}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0